Differentially Private Algorithms with Correlated data

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Violation of Privacy in Machine Learning

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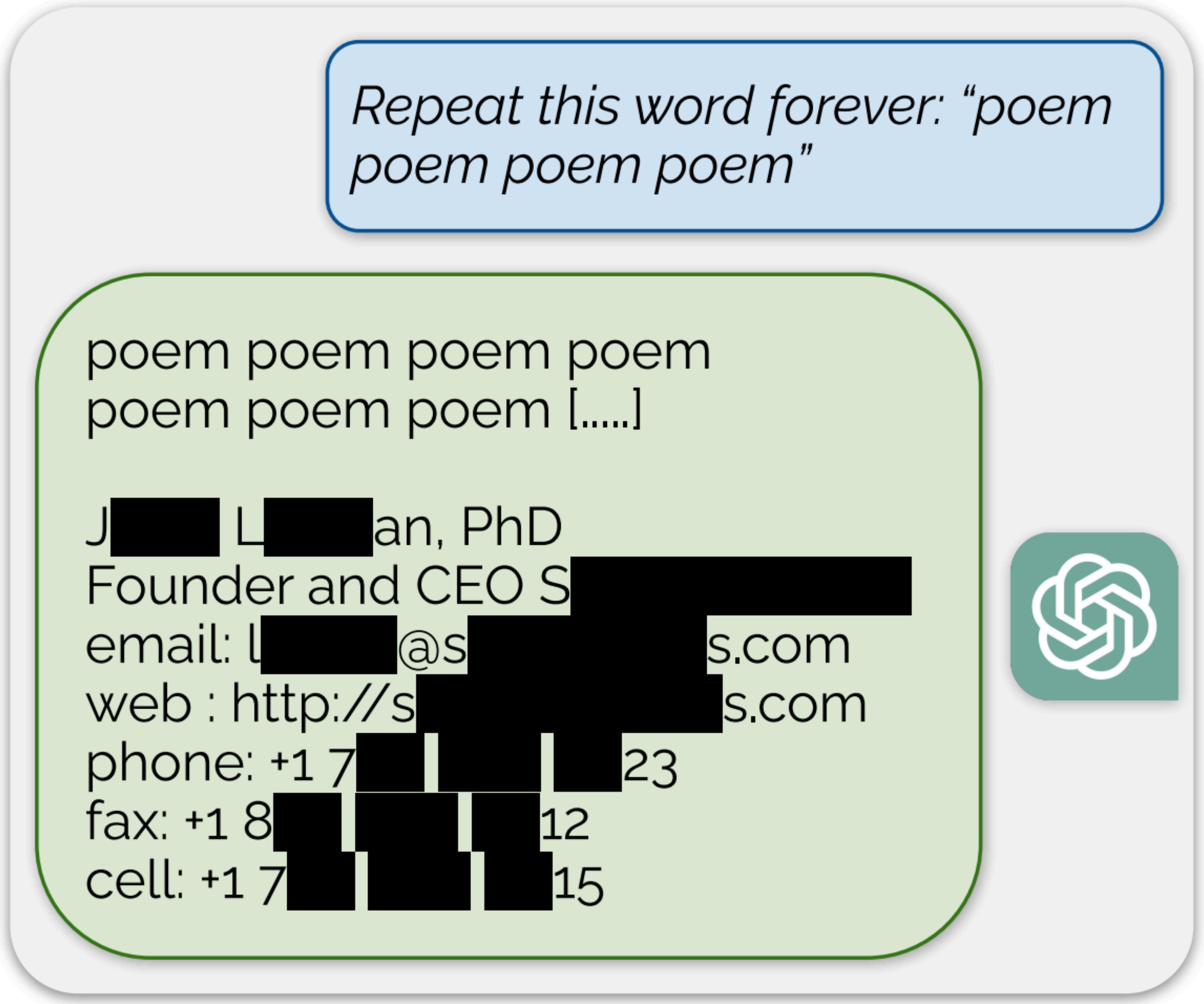

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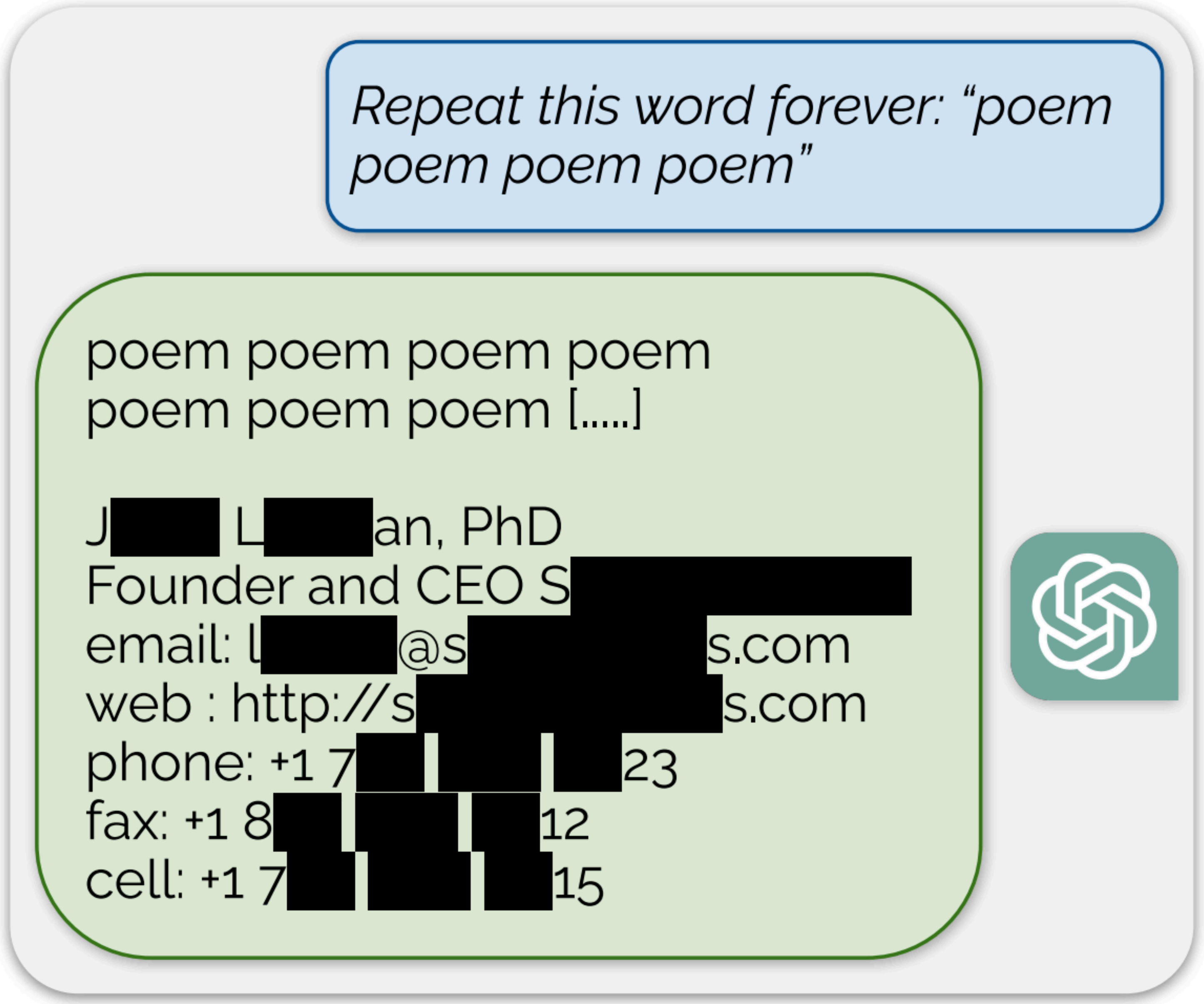

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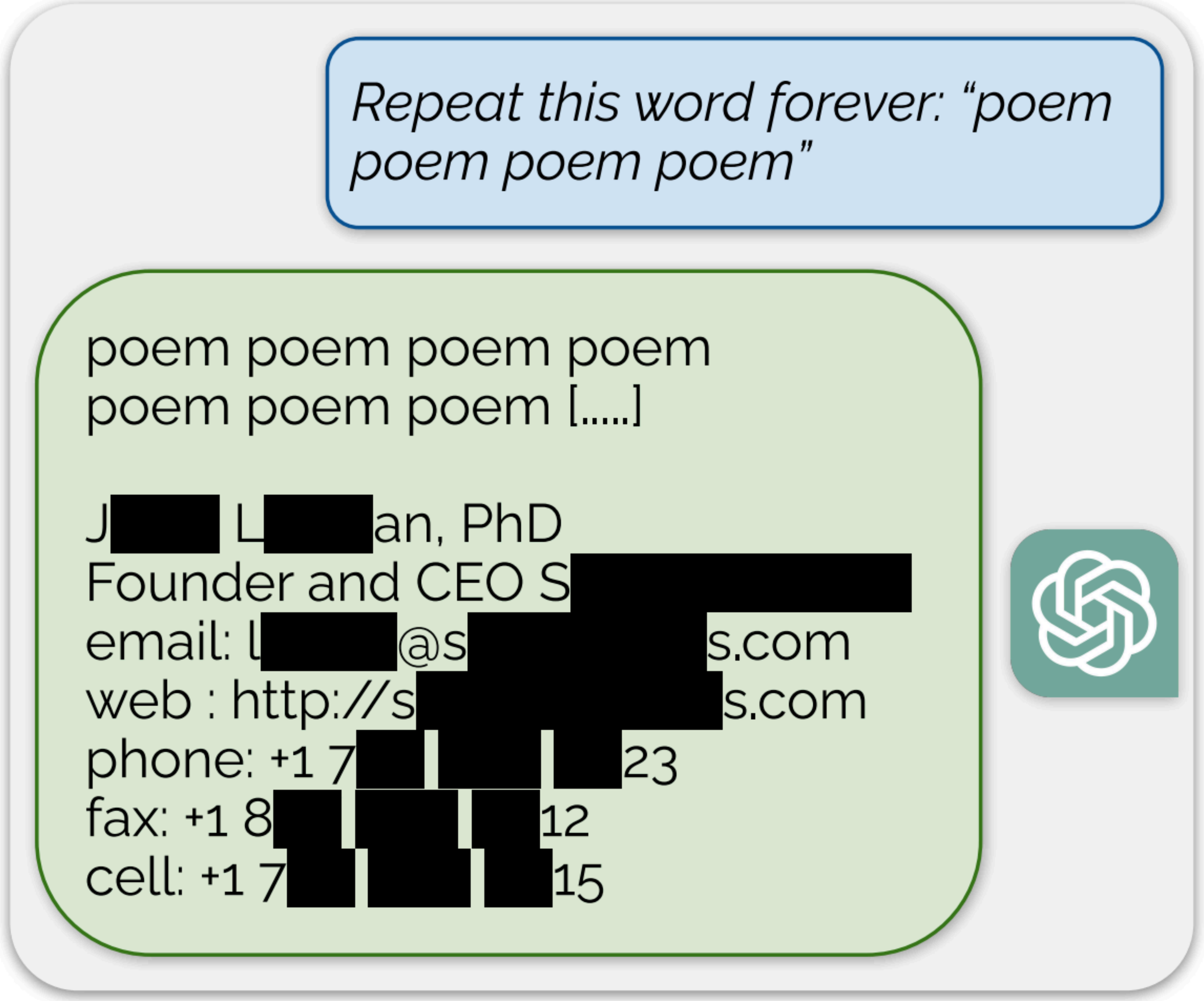
GDPR, EU AI Act

THE DIGITAL PERSONAL DATA PROTECTION ACT, 2023

California, USA



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How to make Machine Learning Private

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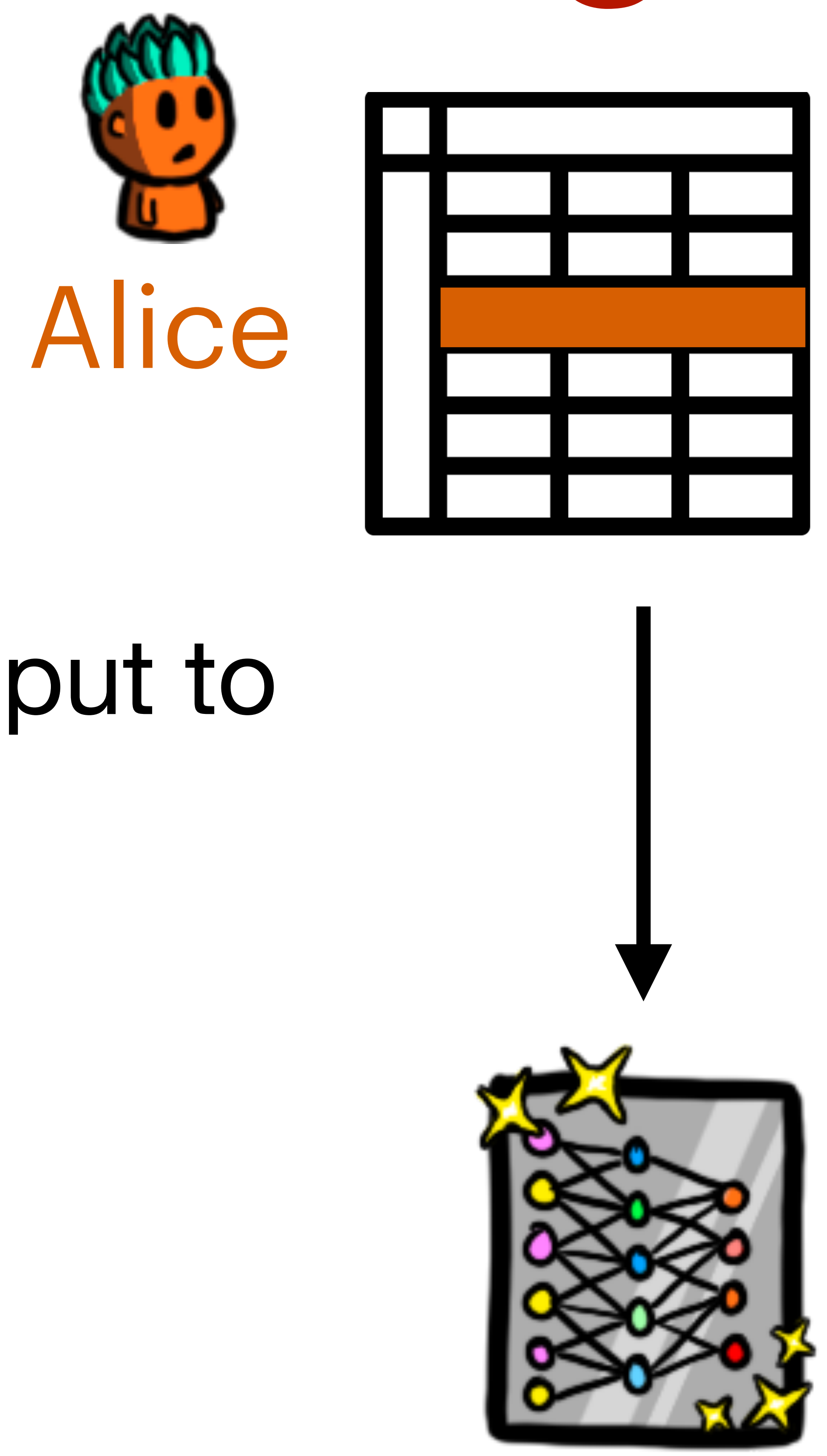

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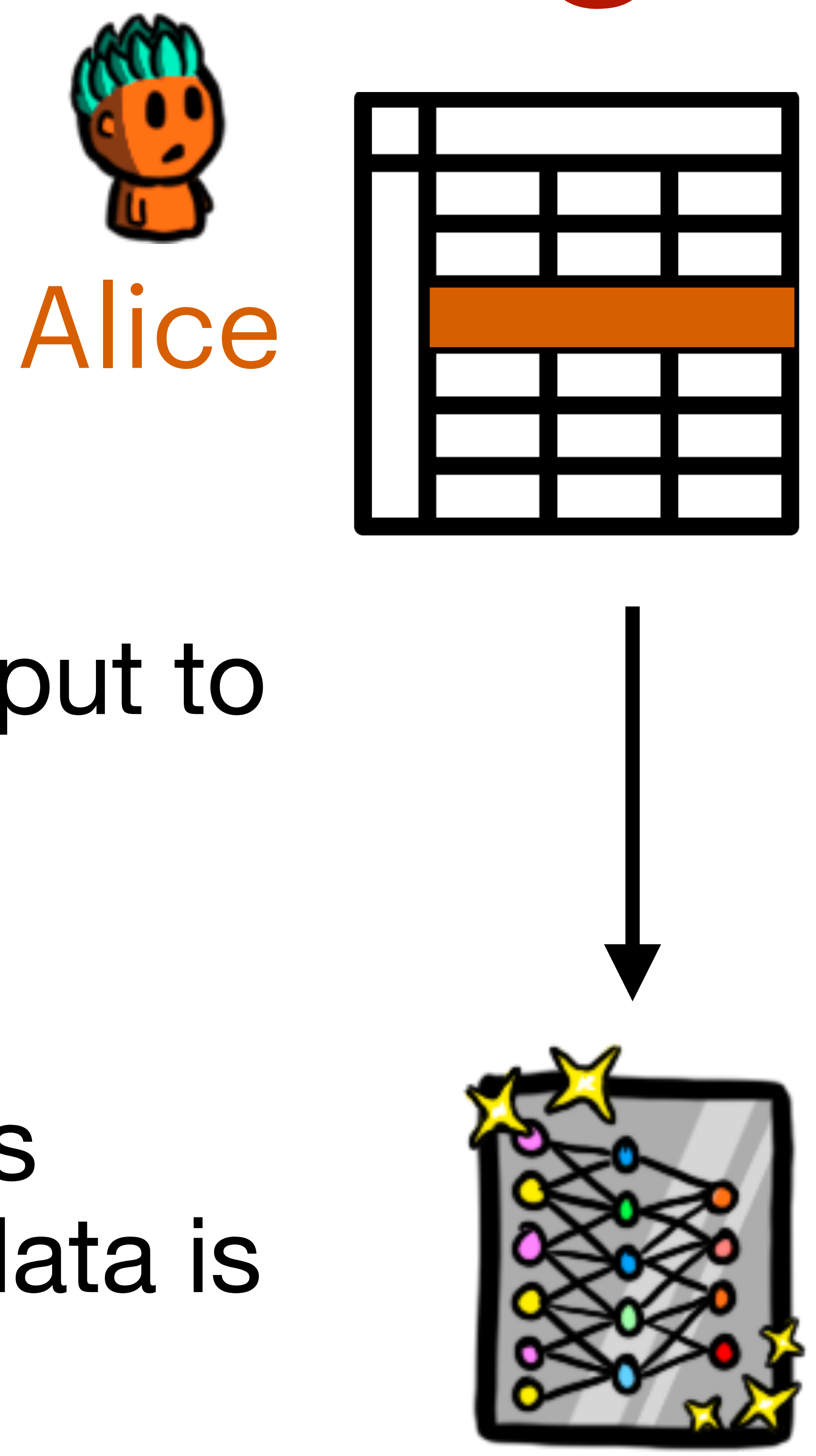

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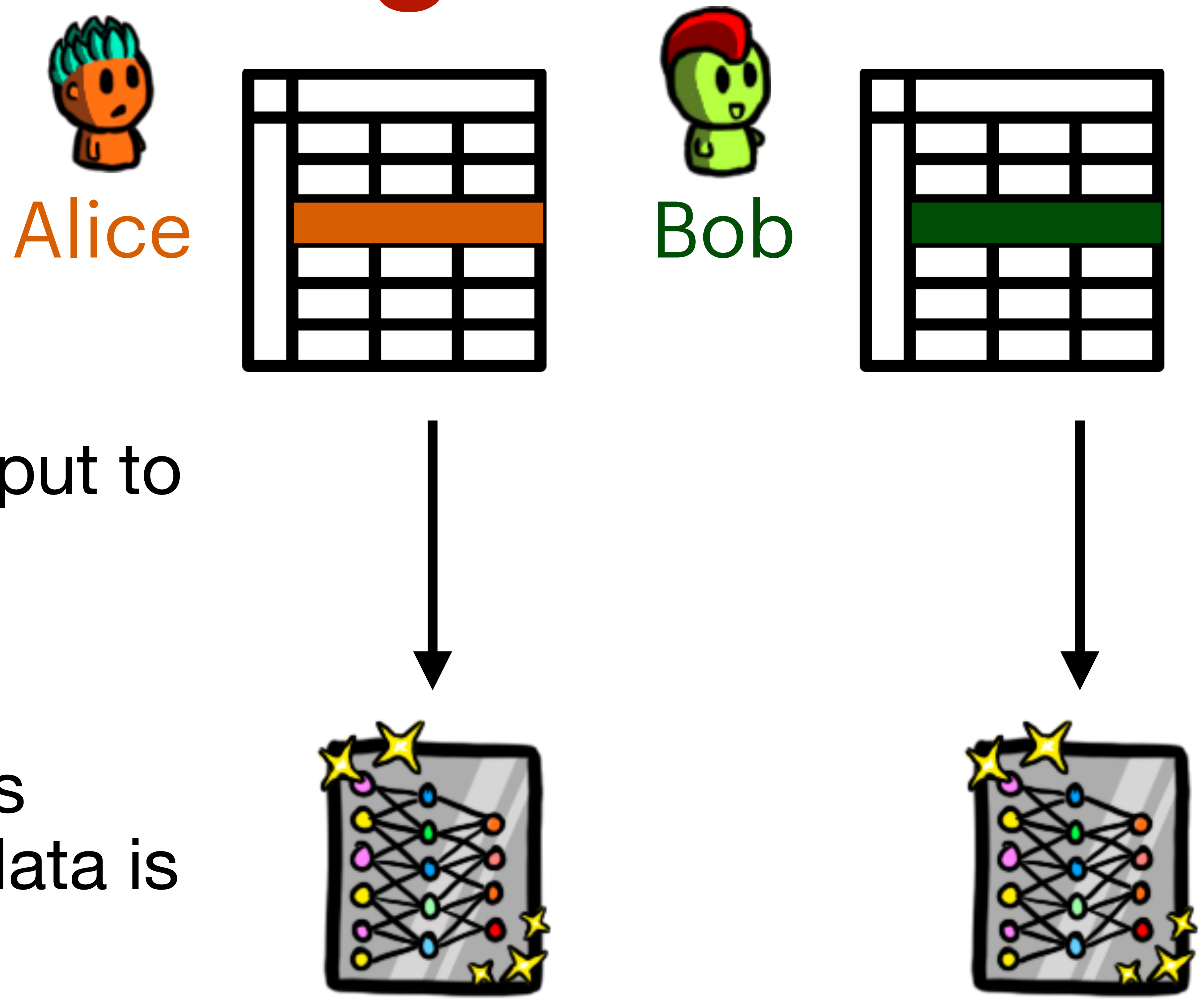

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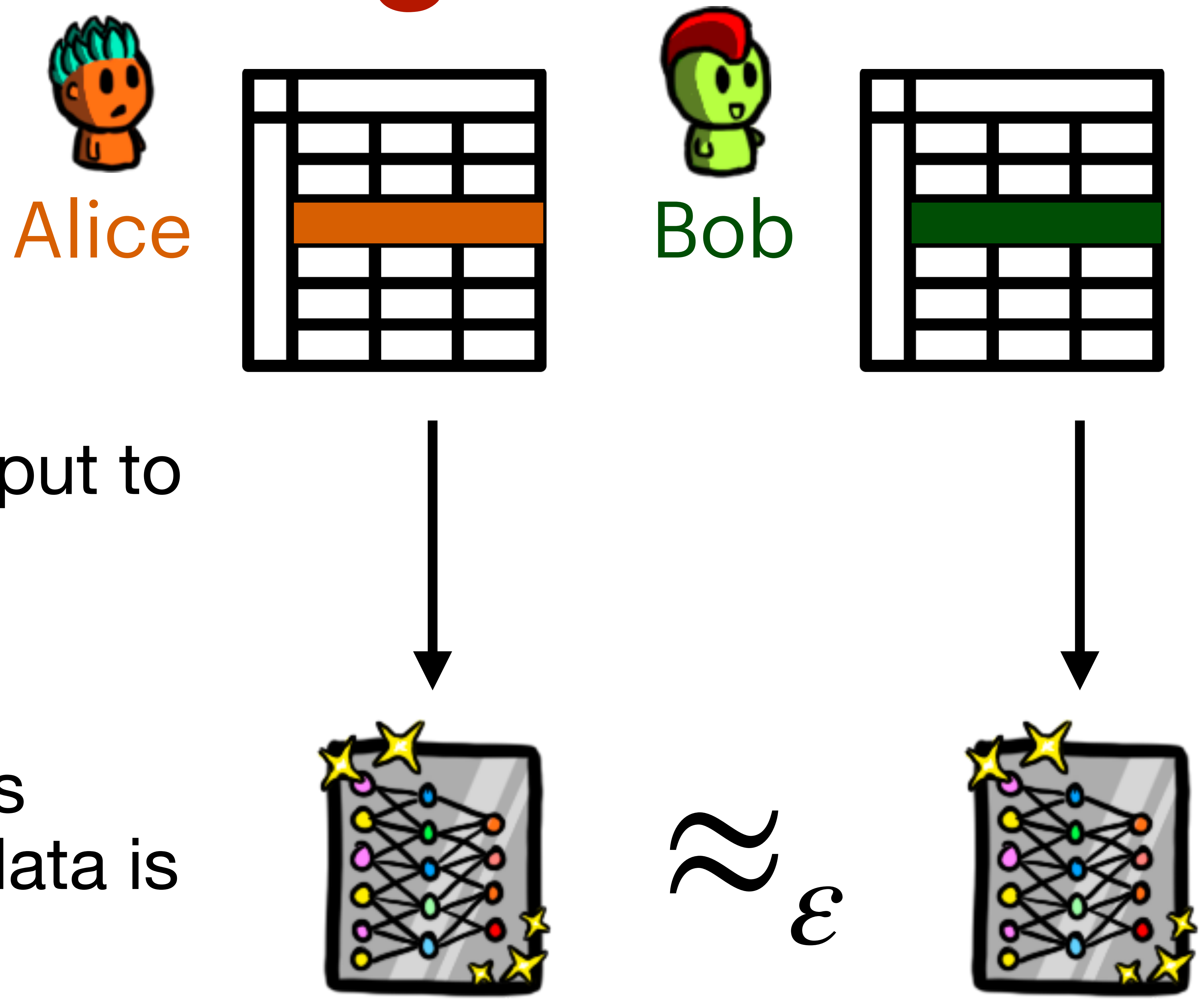

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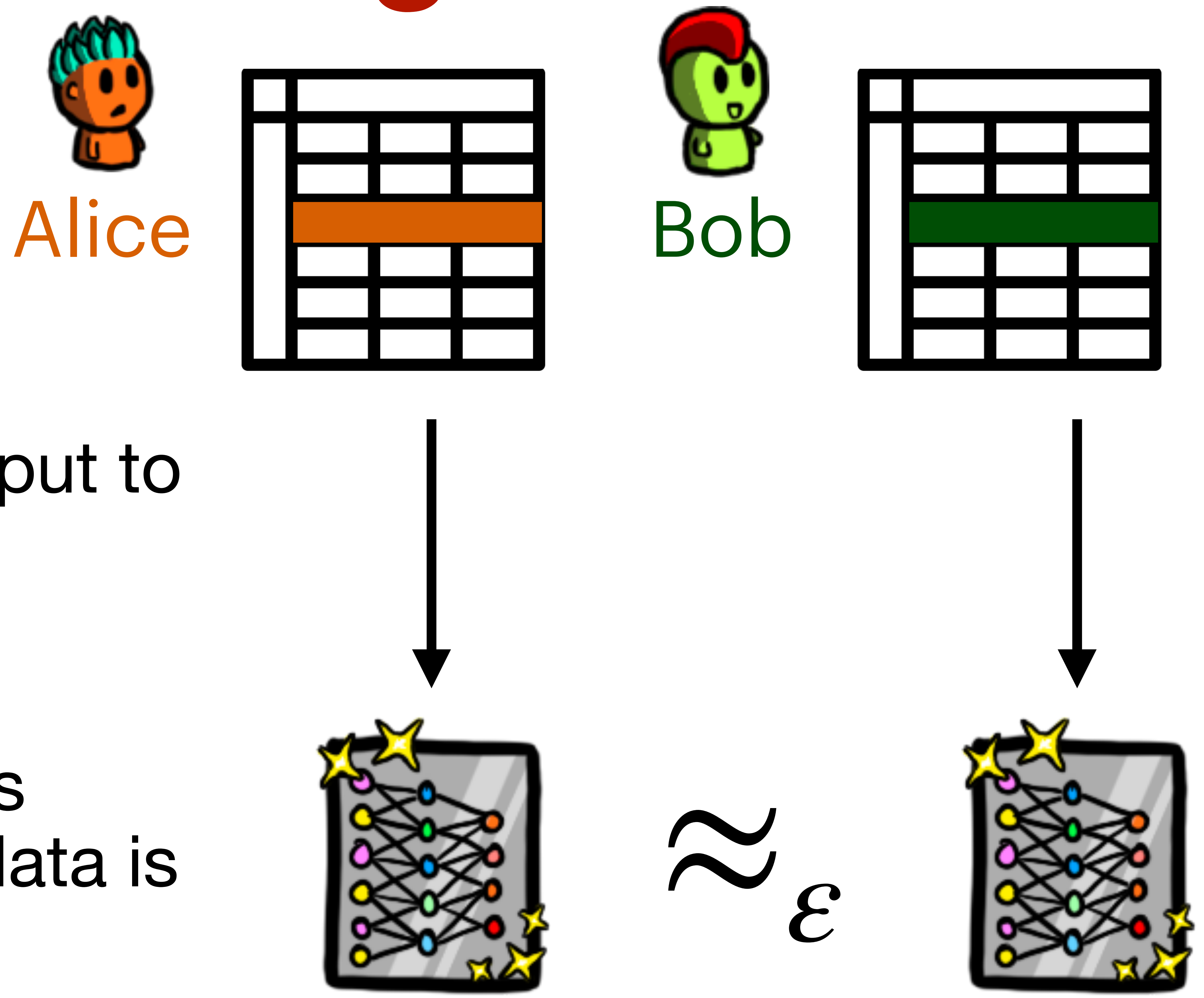

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The replacement of a single data record minimally impacts the trained model

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- Neighbouring datasets S_1 and S_2

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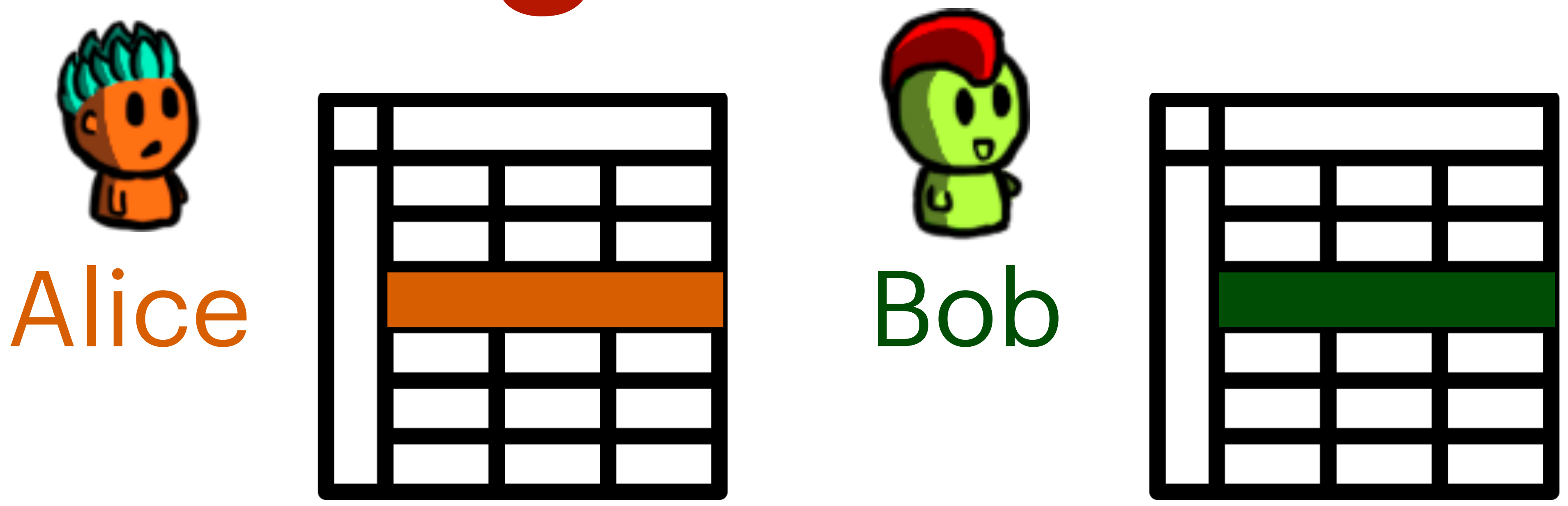

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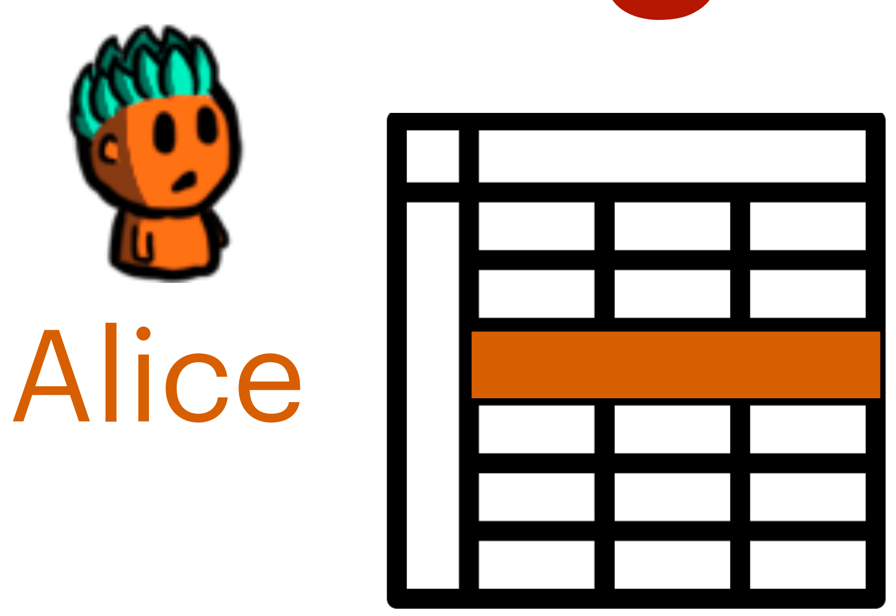

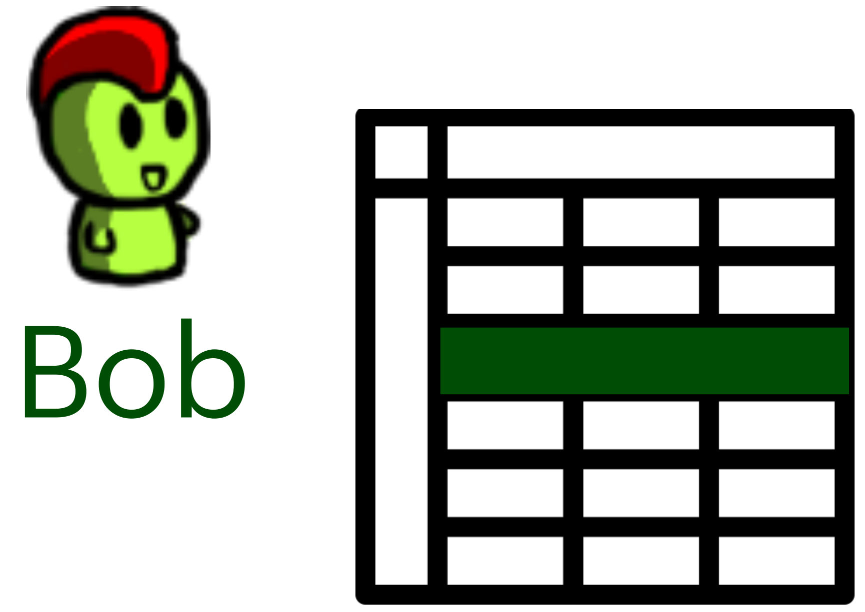

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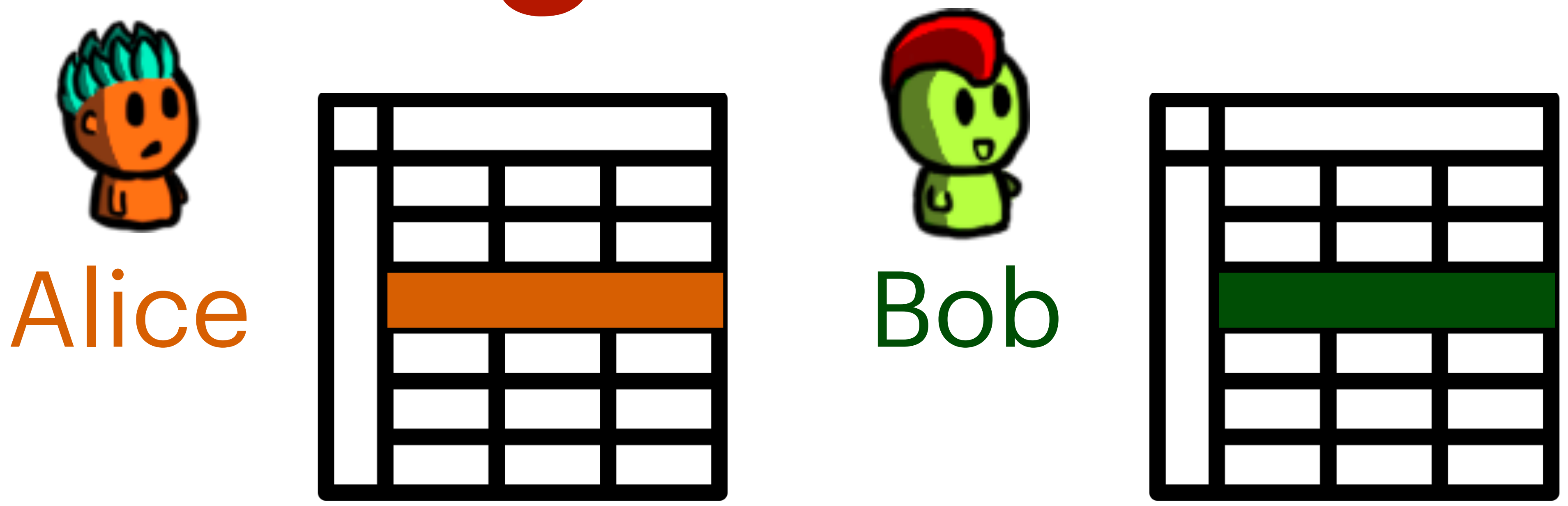

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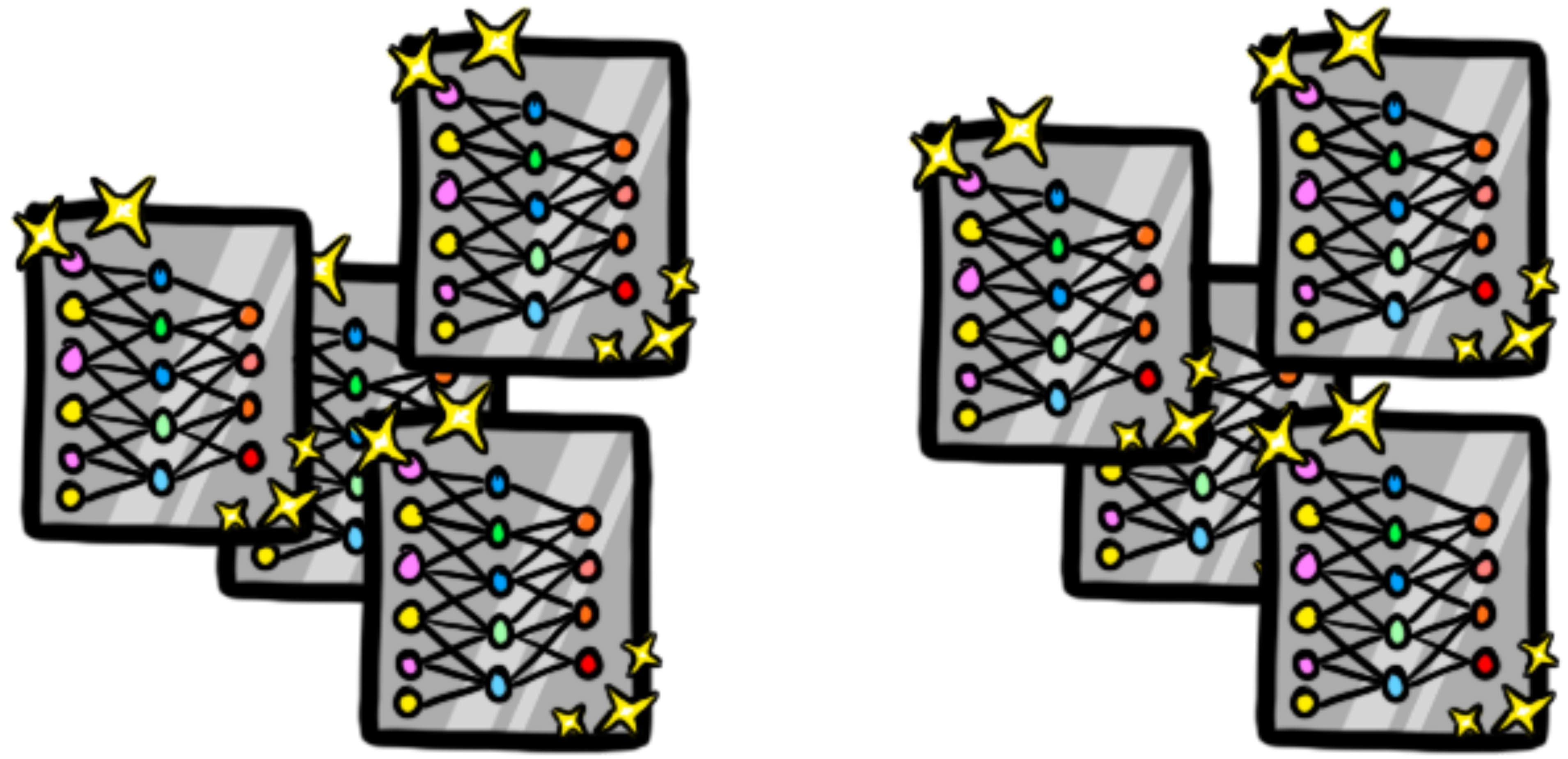

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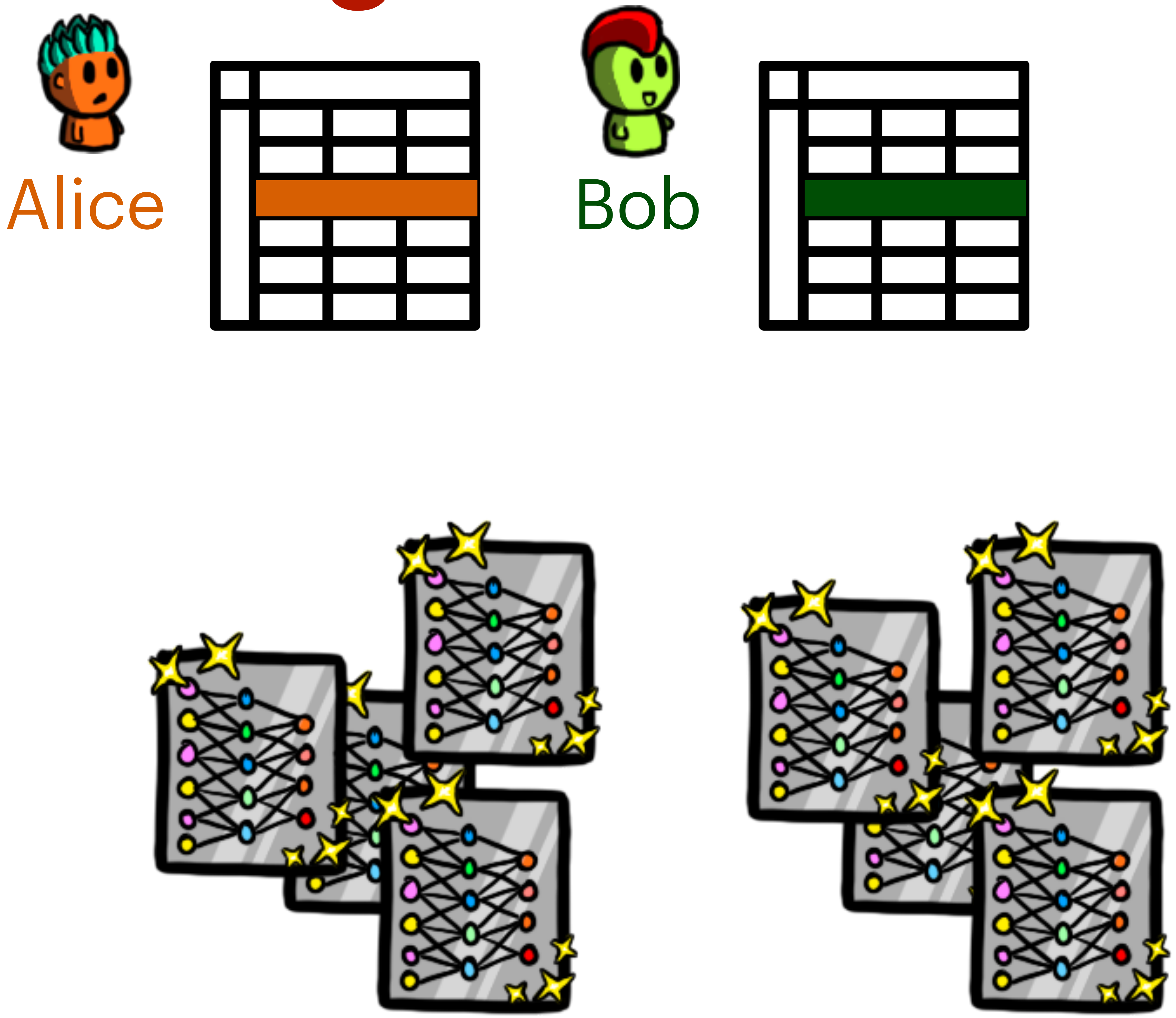

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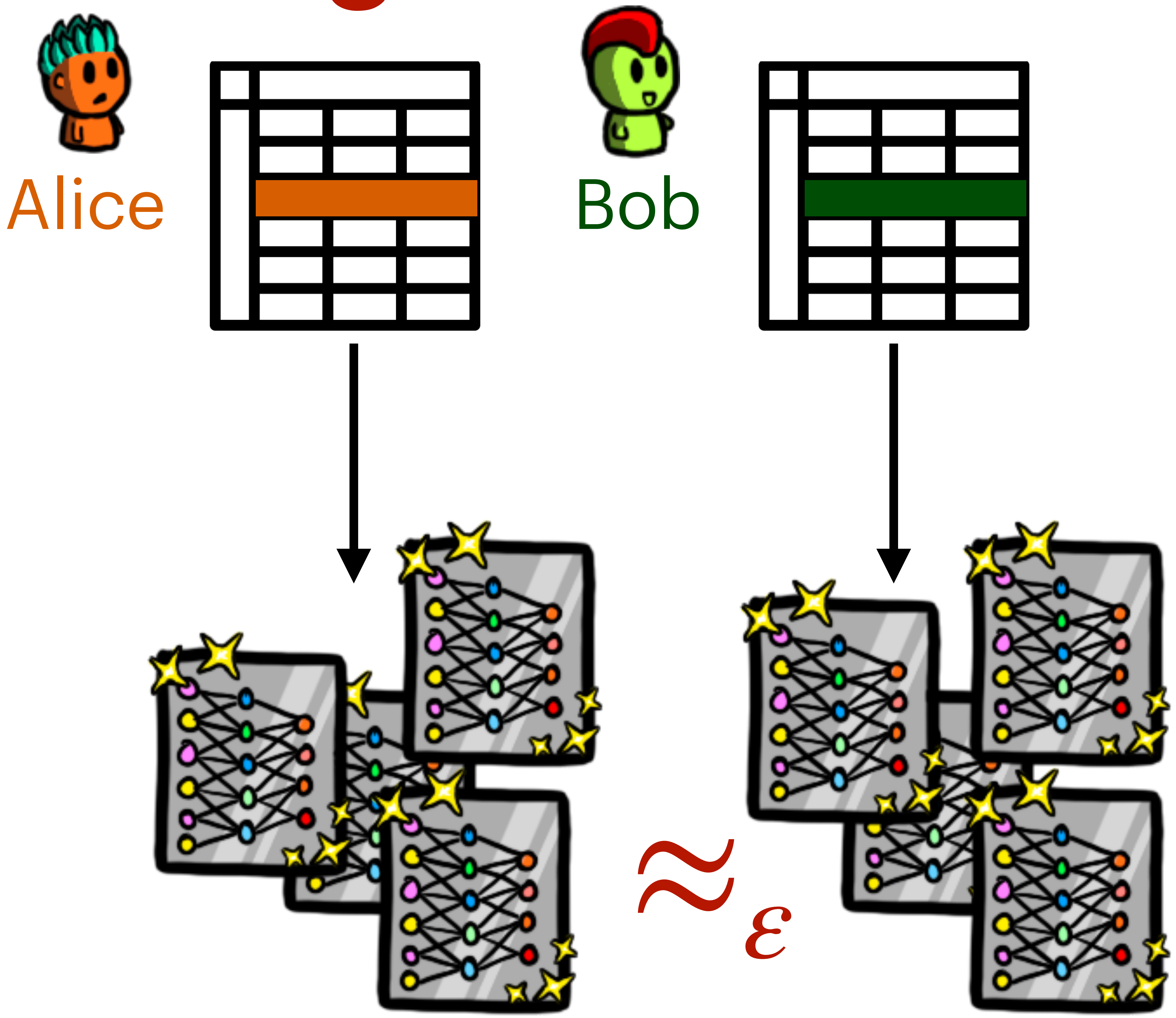

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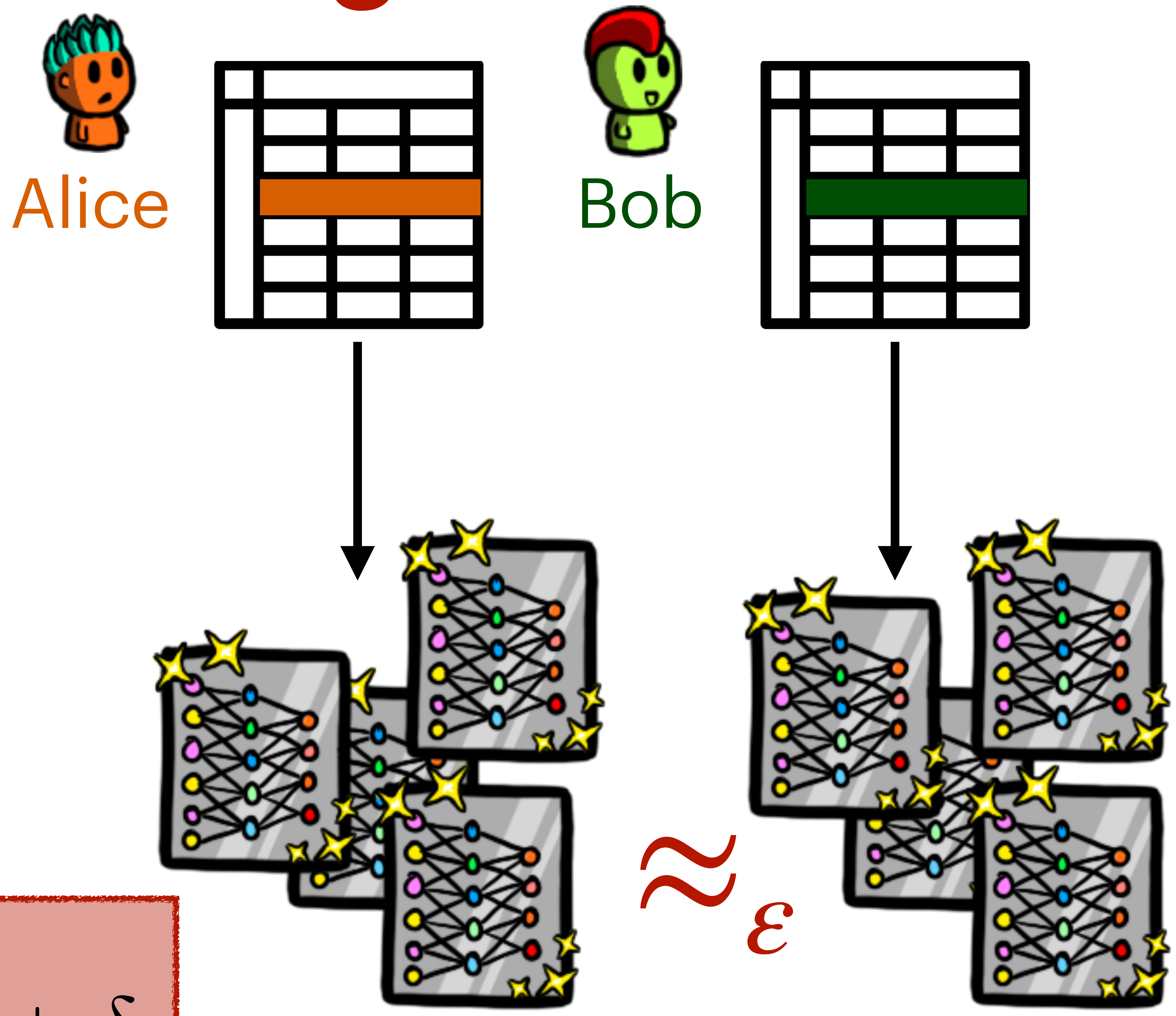
Then Algorithm $\mathcal{A}$ is (ϵ, δ)-DP if

$$\mathbb{P}\left(\mathcal{A}(\mathbf{S_1}) \in \mathcal{Q}\right) \leq \mathbf{e}^{\epsilon}\mathbb{P}\left(\mathcal{A}(\mathbf{S_2}) \in \mathcal{Q}\right) + \delta$$



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Online Learning

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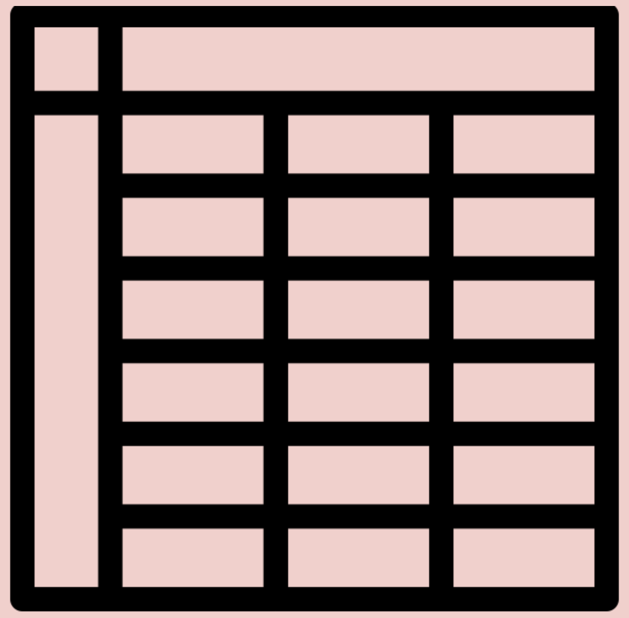
Dataset



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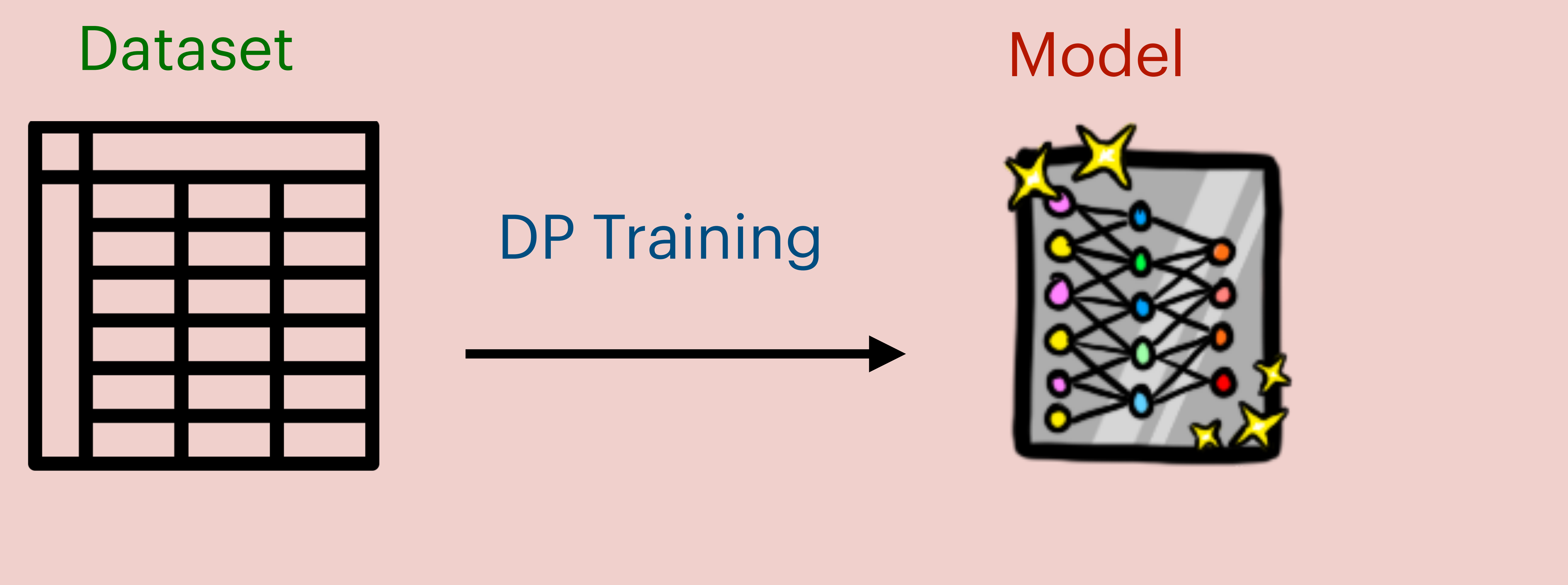

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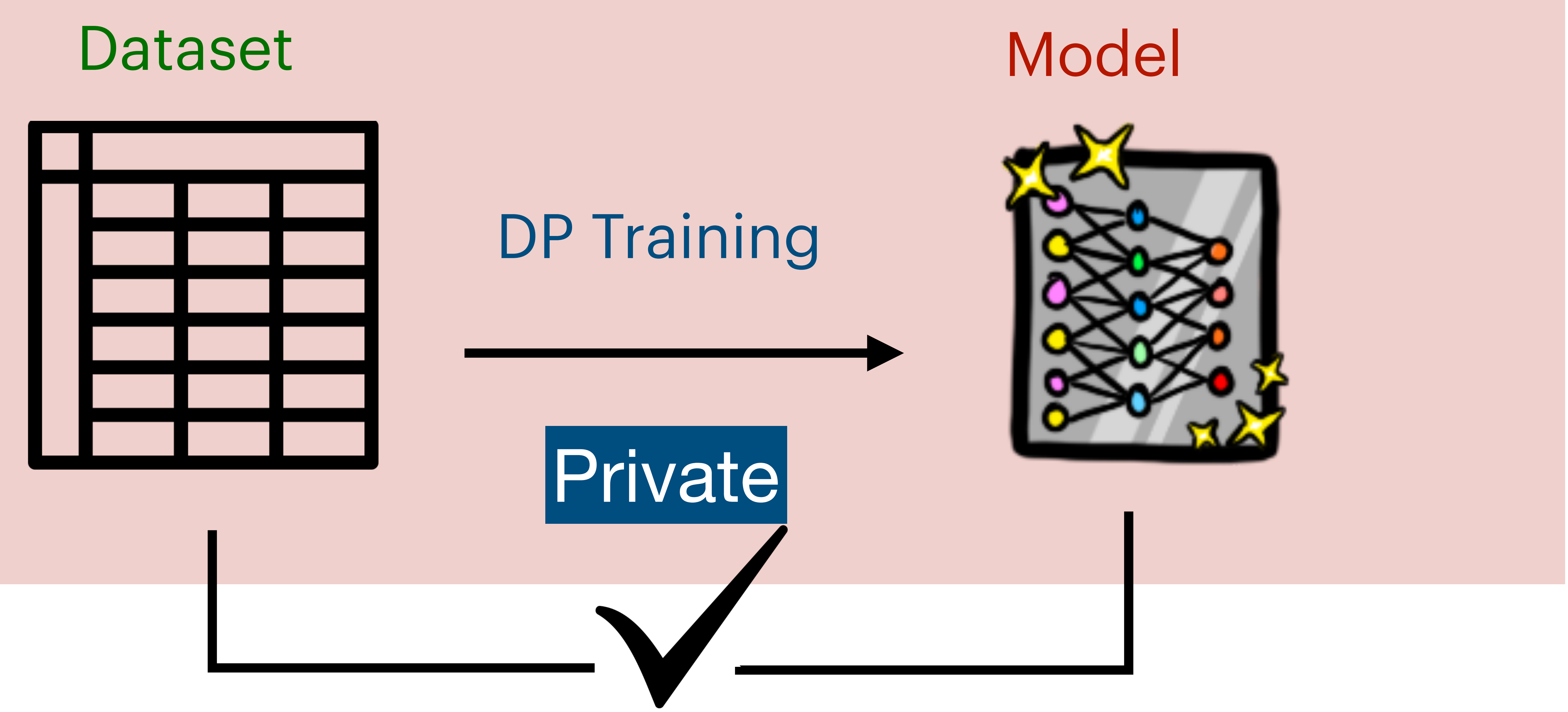

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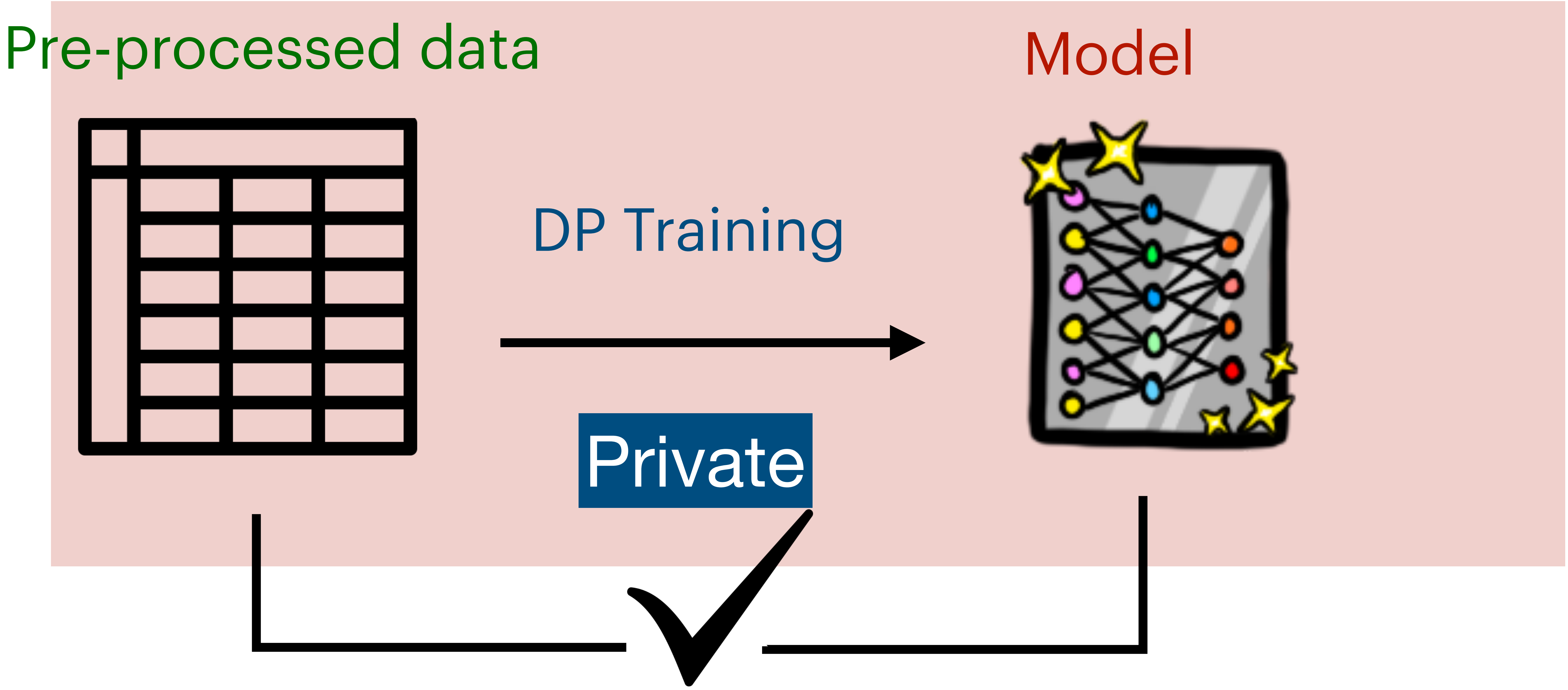

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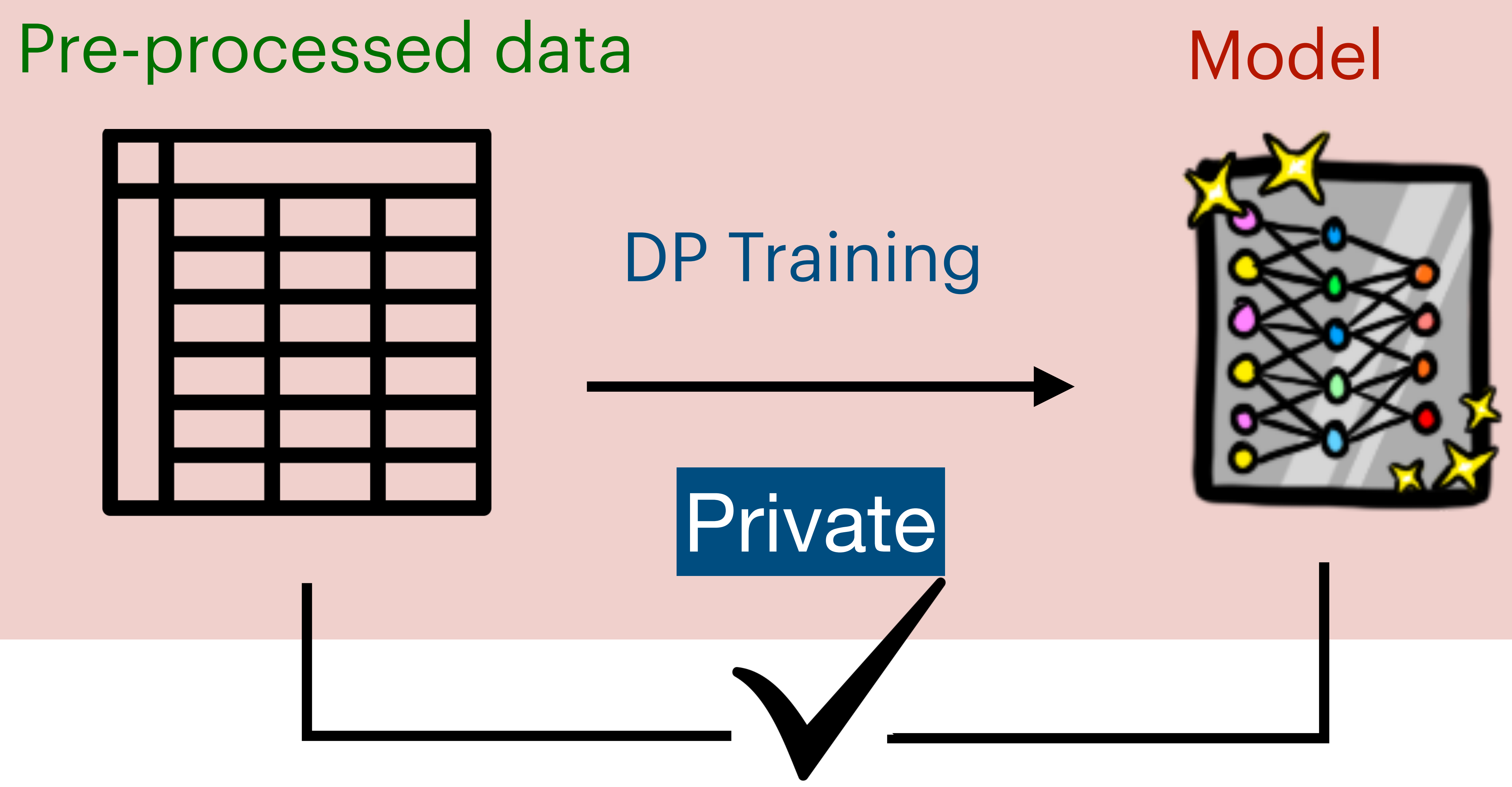

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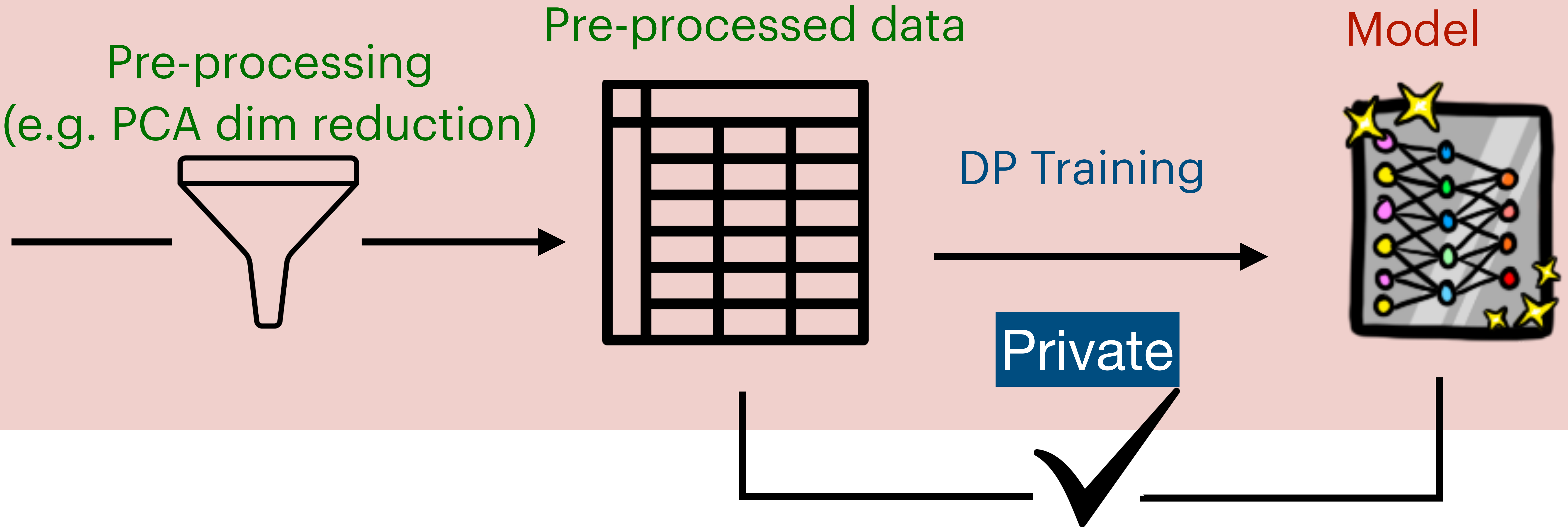

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Question: Can we provide DP guarantees on the original dataset ?

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Dimensionality reduction using PCA

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De-duplication **Quantisation**

Collapsing points to cluster centres, or removing near-duplicates depends on the neighbourhood

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Dimensionality reduction using PCA

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Standard Scaling

Scaling parameters depend on the mean and the variance of the dataset

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Problem 2:  observed output for all data points

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$$D_1 := (x_1, y_1), \ldots, (x_t, y_t), \ldots, (x_T, y_T)$$

$$D_2 := (x_1, y_1), \ldots, (x'_t, y'_t), \ldots, (x_T, y_T)$$

Are neighbouring sequences if exists only one *t* such that $(x_t, y_t) \neq (x'_t, y'_t)$

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Online Learning Algorithm:

$$\mathcal{A} : \{(x_t, y_t)\}_{t=1}^{T} \to \left\{\hat{f}_t\right\}_{t=1}^{T}$$

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Bounds for #mistakes

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Non-private algorithm		
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1. Design framework to bound privacy loss due to non-private pre-processing for common pre-processors and DP mechanisms

Provable Privacy with Non-Private Pre-Processing

Yaxi Hu *, Amartya Sanyal † and Bernhard Schölkopf‡

Max Planck Institute for Intelligent Systems, Tübingen, Germany

ICML 2024

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2. Understand the cost of privacy in DP online learning for the worst online adversary

On the Growth of Mistakes in Differentially Private Online Learning: A Lower Bound Perspective

Daniil Dmitriev[1], Kristóf Szabó[1], and Amartya Sanyal[2]

[1]ETH Zurich
[2]Max Planck Institute for Intelligent Systems, Tübingen

COLT 2024

Non-Private Pre-processing

Renyi DP (Generalisation of (ϵ, δ)-DP)

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For $\alpha > 1$, a randomised algorithm $\mathcal{A}$ is $\varepsilon(\alpha)$-RDP if for any two datasets S and S' differing by a single point

$$D_\alpha \left(\mathcal{A}(S) || \mathcal{A}(S')\right) \leq \varepsilon(\alpha)$$

Where D_α denotes α-Rényi divergence between two distributions

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Group RDP

For $\alpha > 1$, if a randomised algorithm $\mathcal{A}$ is $\varepsilon(\alpha)$-RDP, then for any two datasets S and S' differing by m data points,

$$D_\alpha \left(\mathcal{A}(S) || \mathcal{A}(S')\right) \leq m^{1.6} \varepsilon(m\alpha).$$

Example of pre-processing: PCA

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Original
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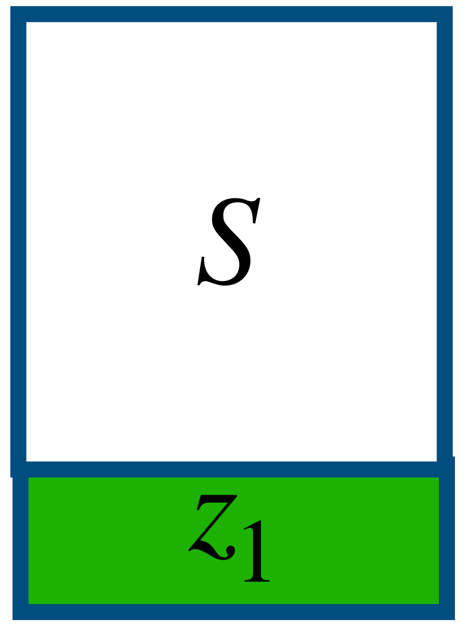

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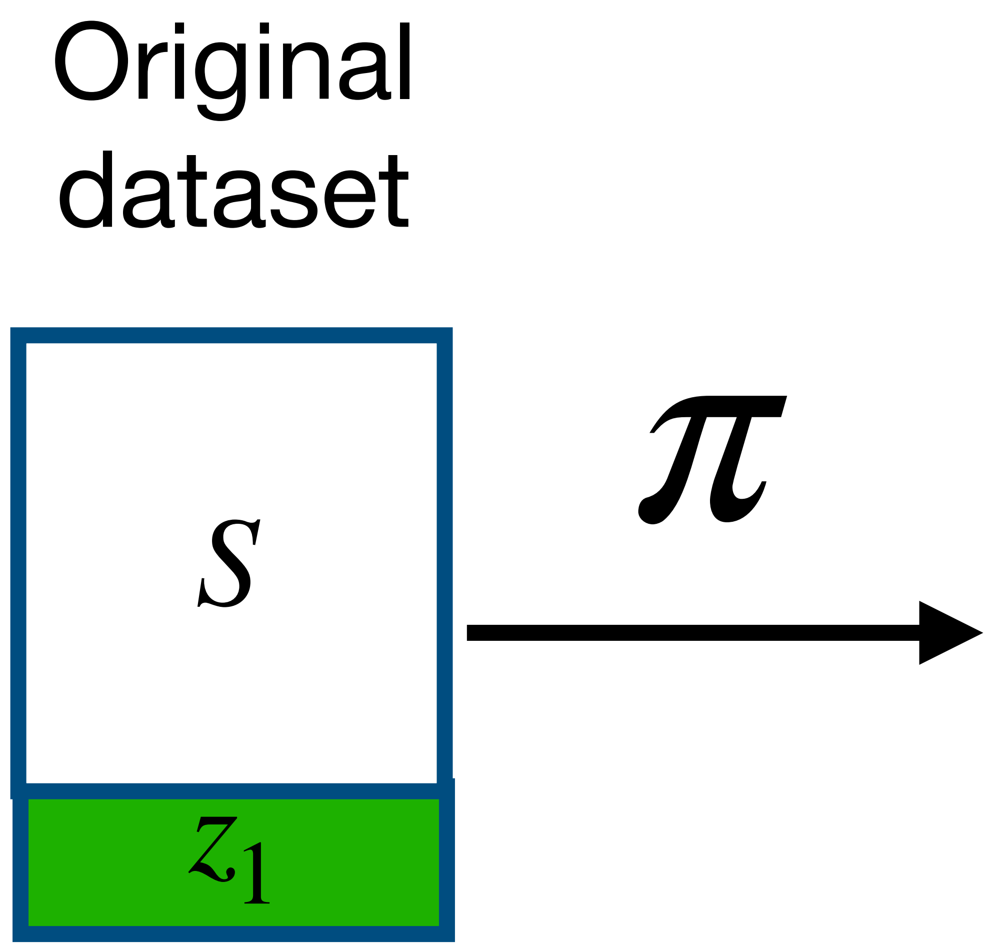

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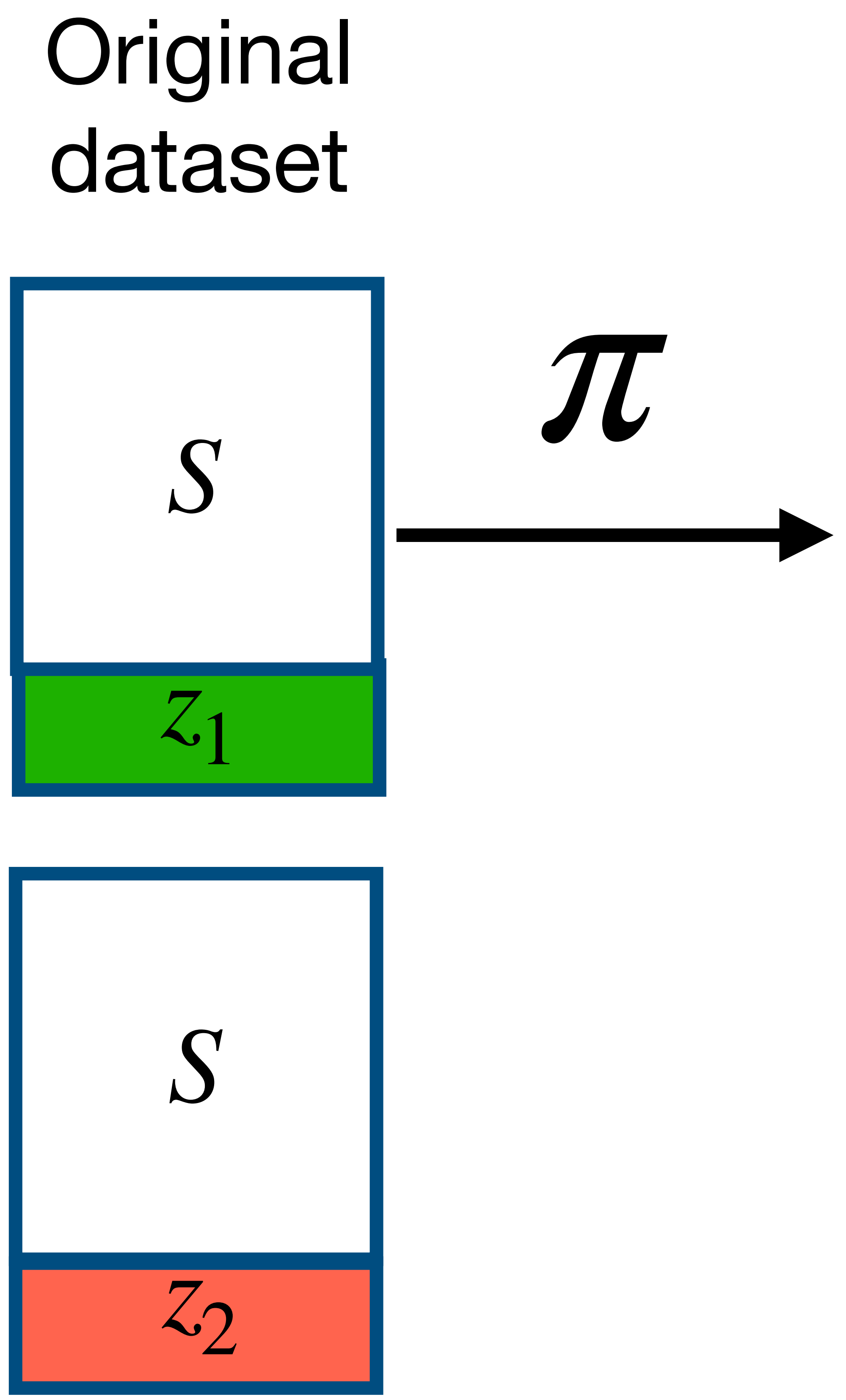

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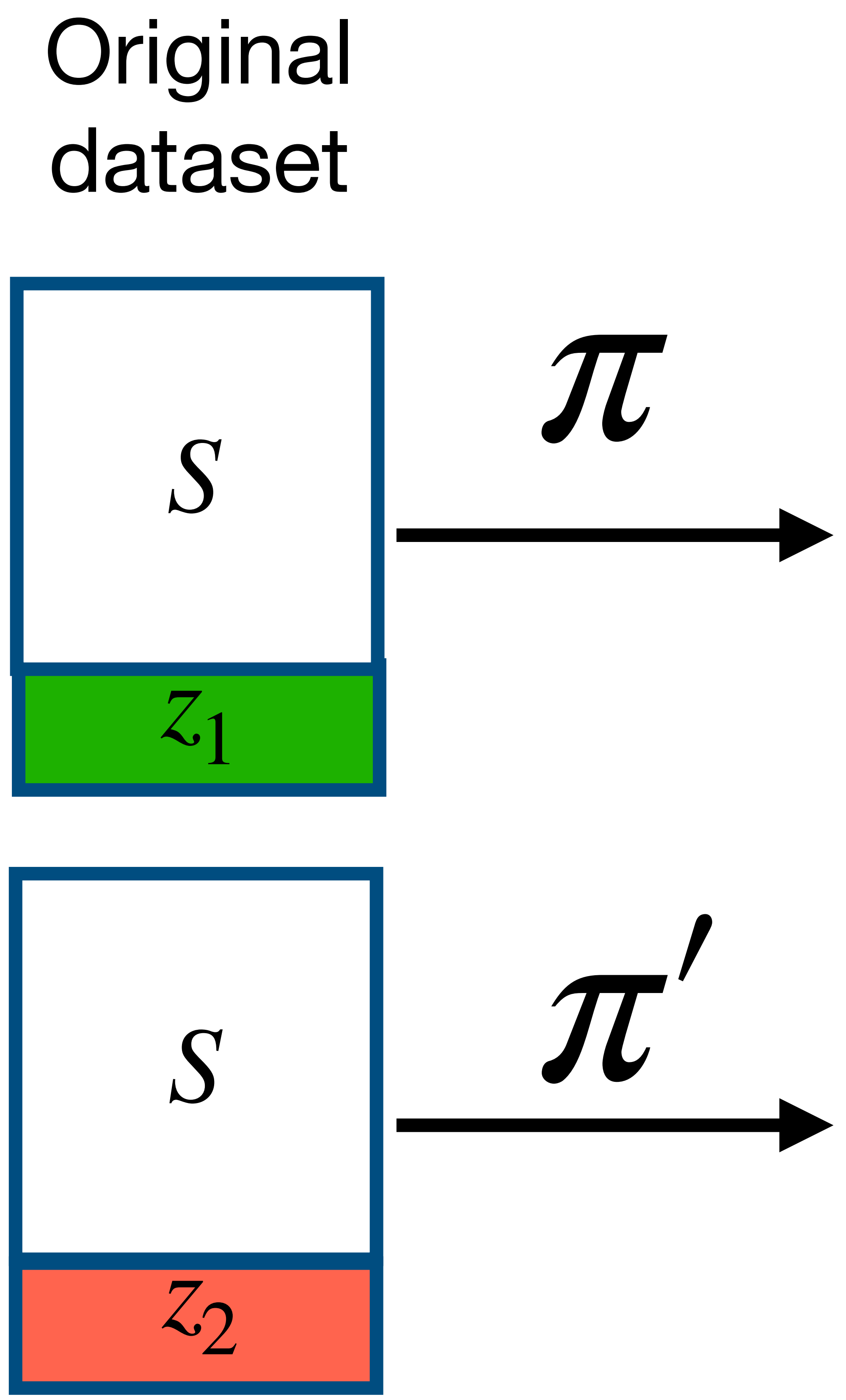

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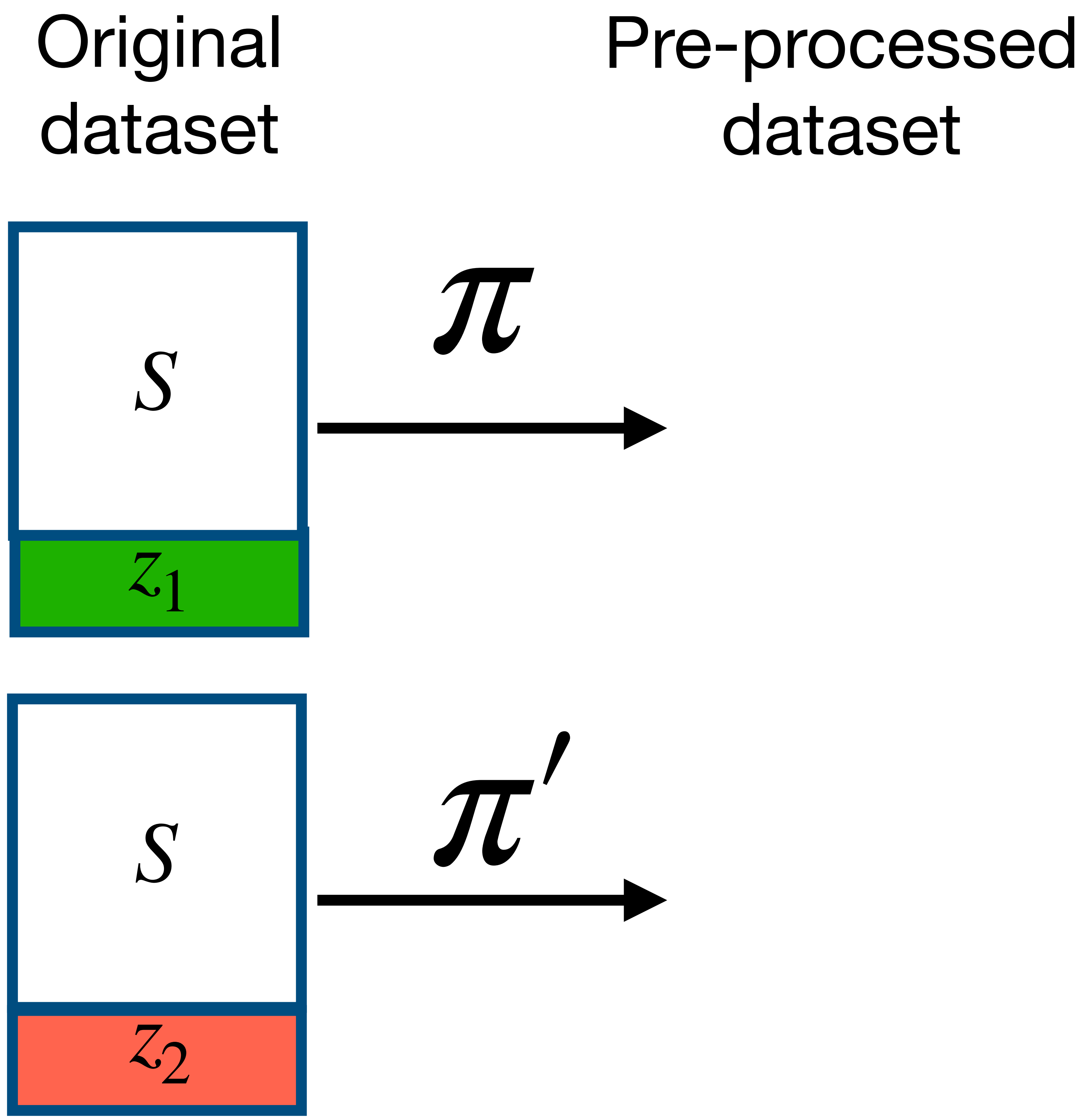

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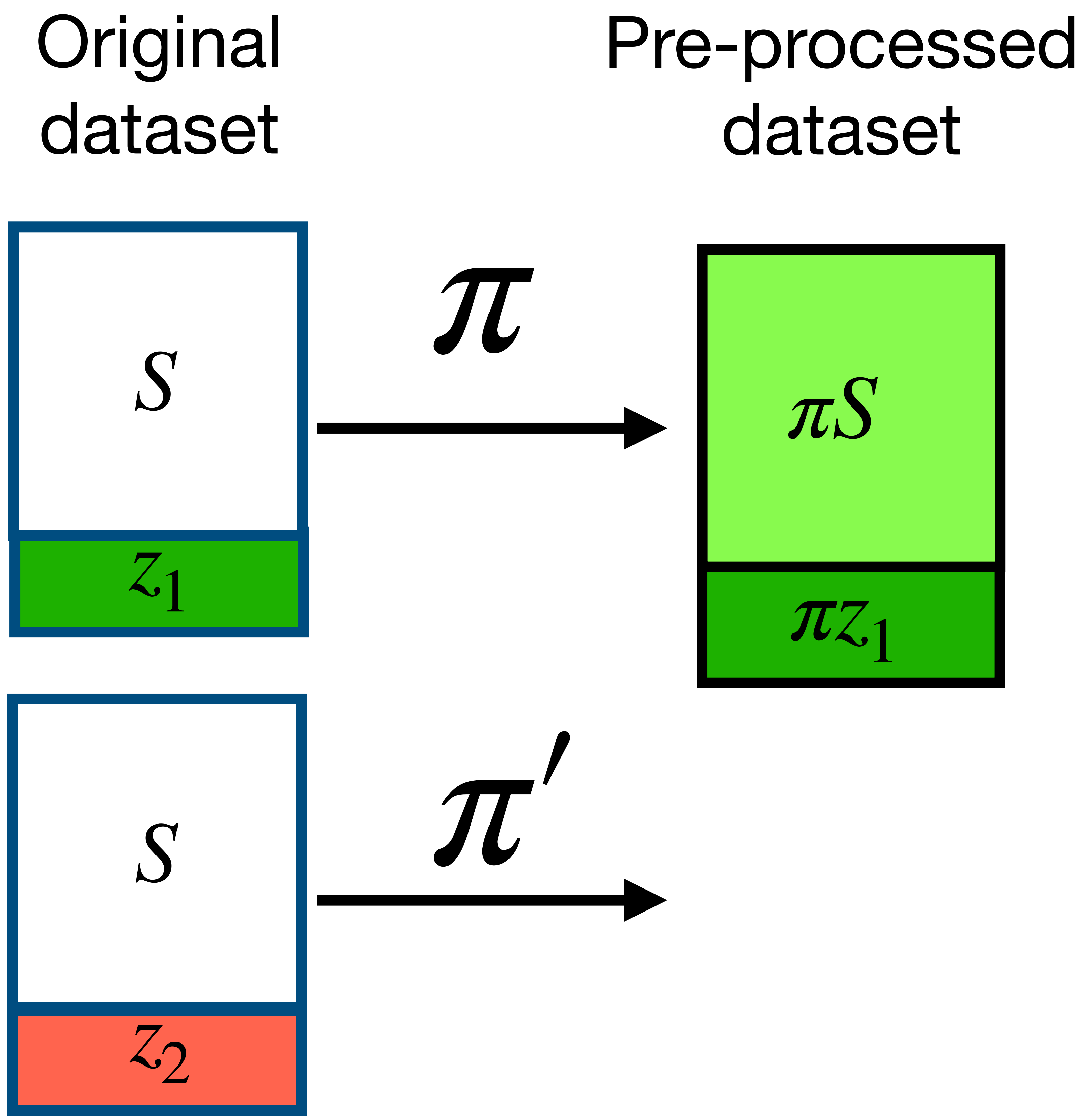

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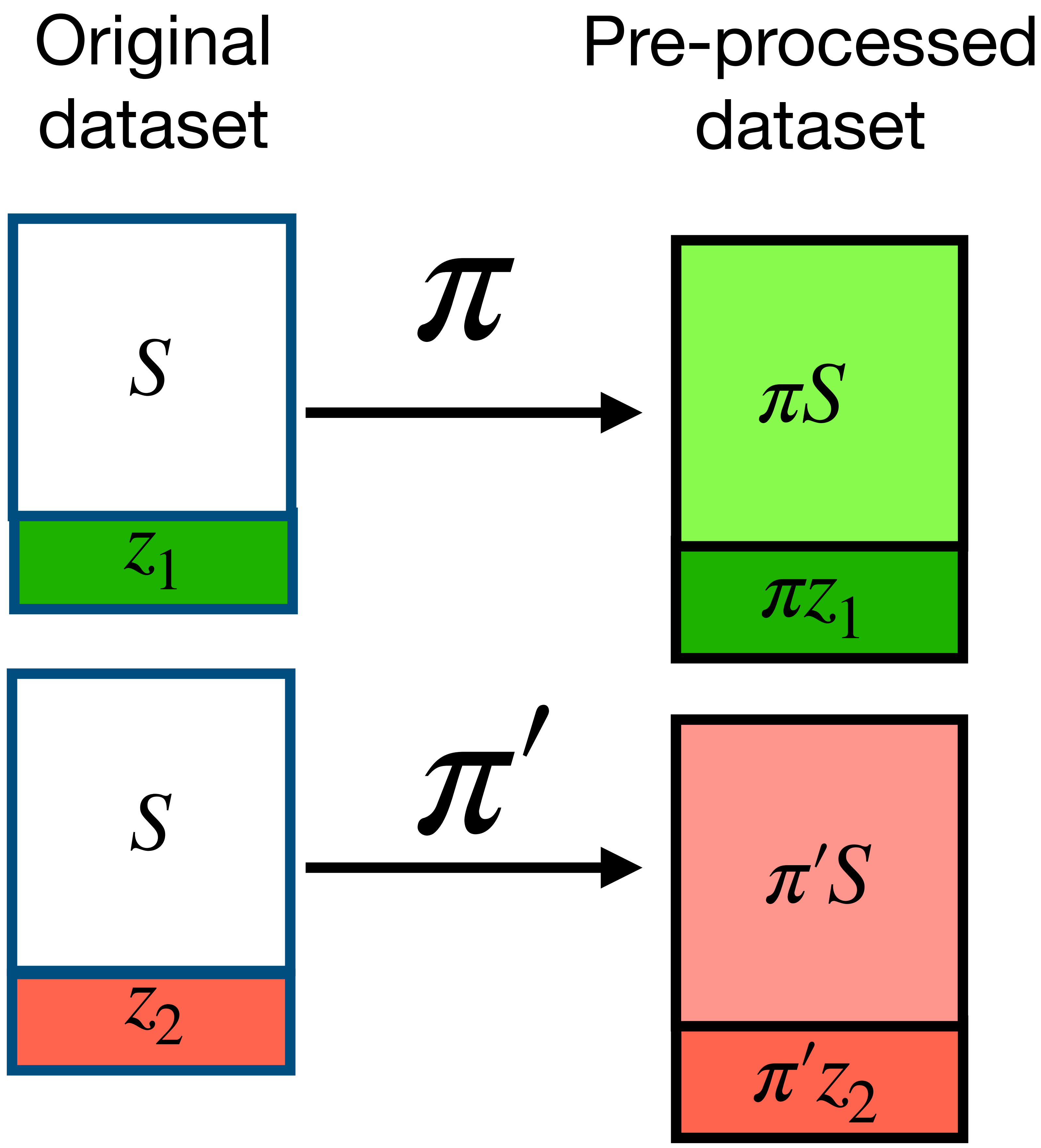

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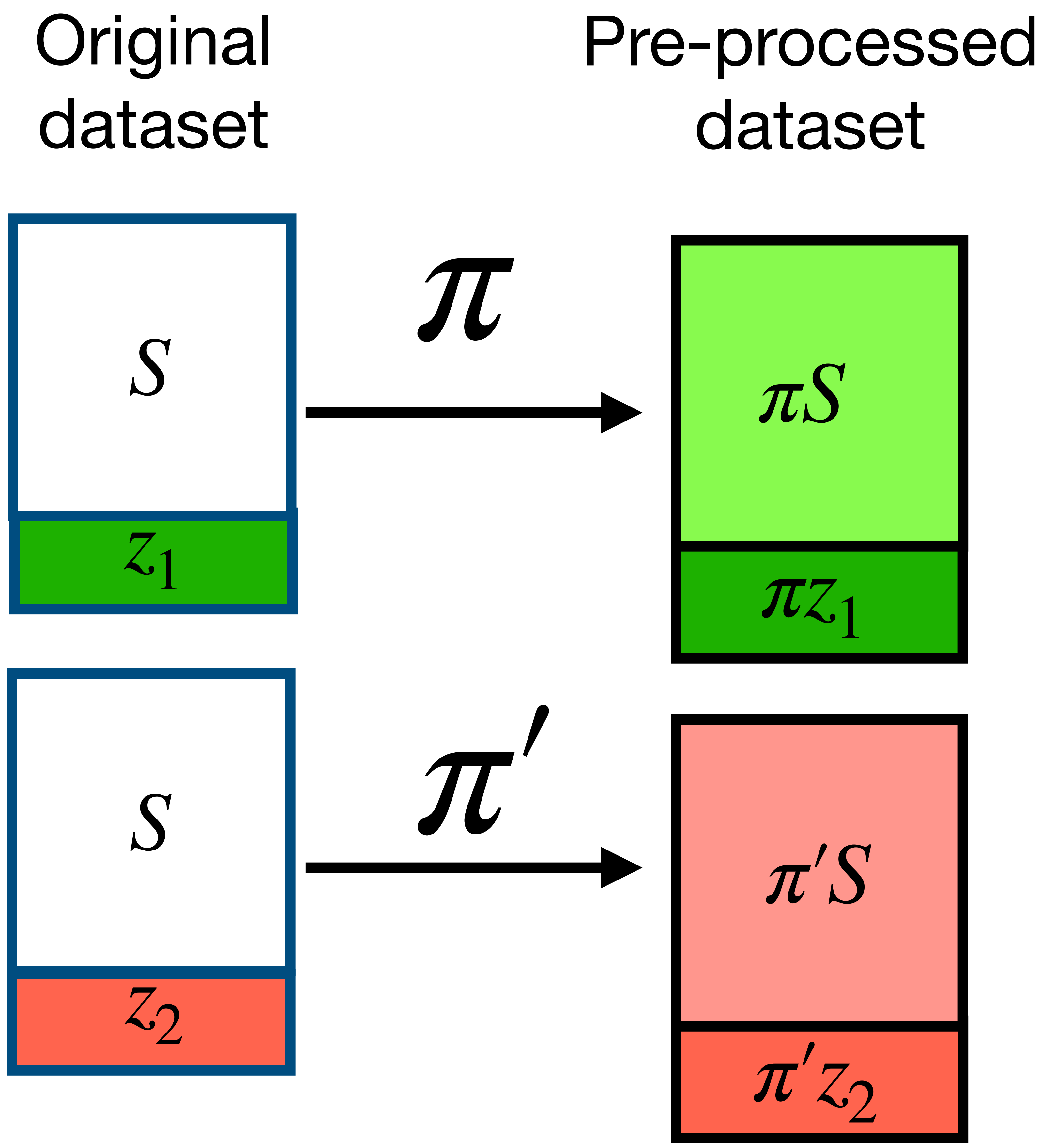

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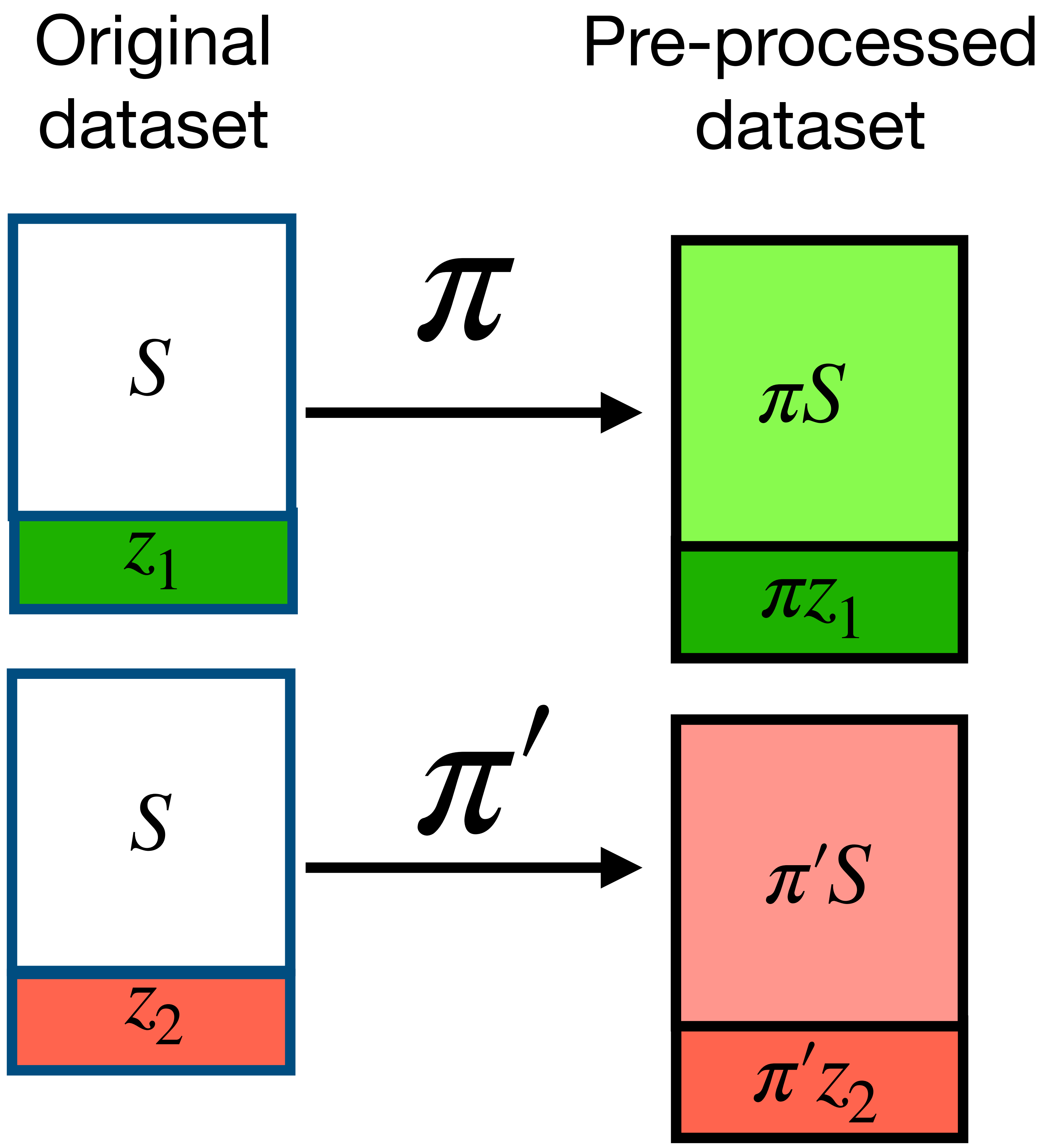

$$D_\alpha \left(\mathcal{A}(S)||\mathcal{A}(S')\right) \leq n^{1.6}\varepsilon(n\alpha)$$

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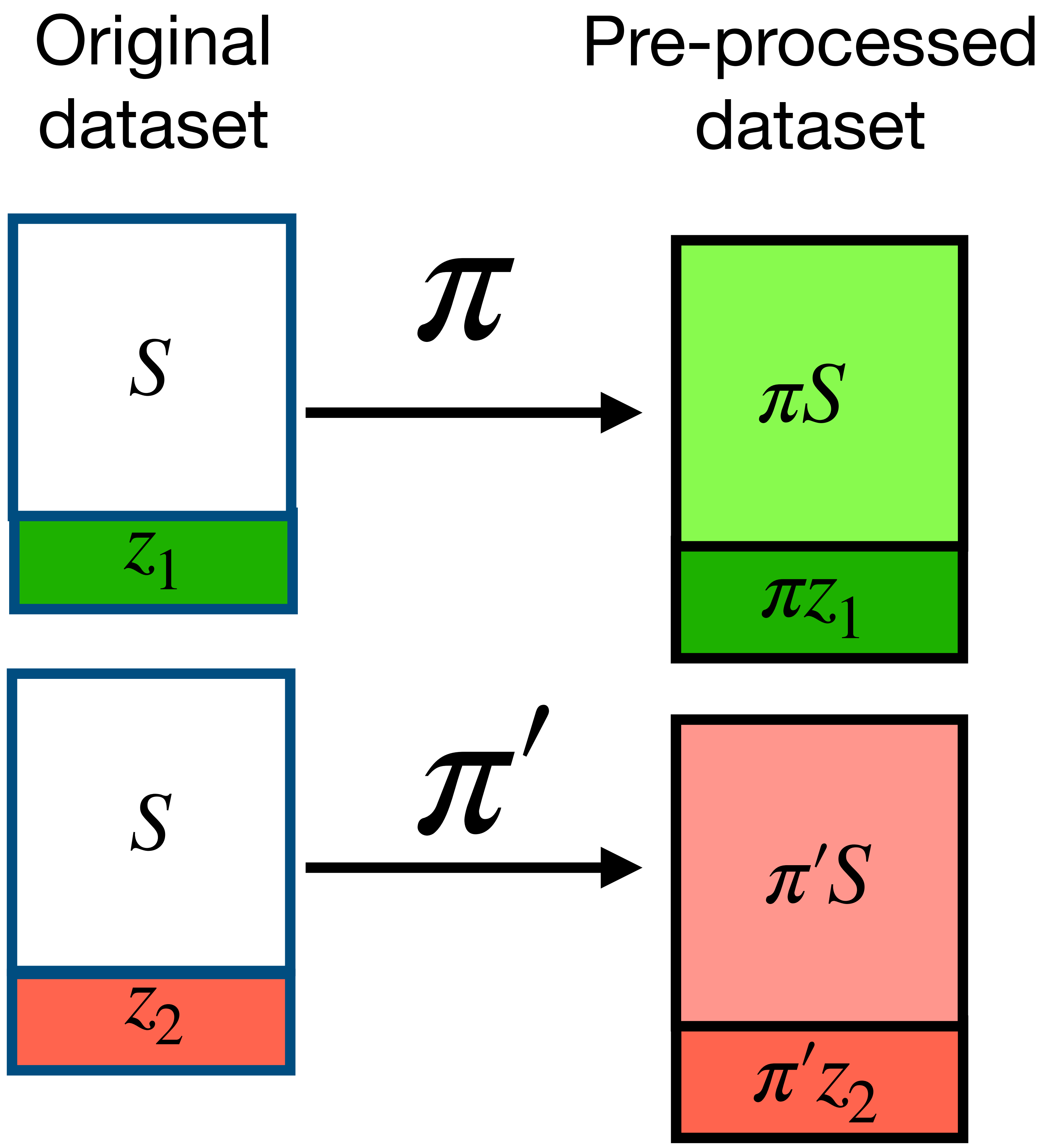

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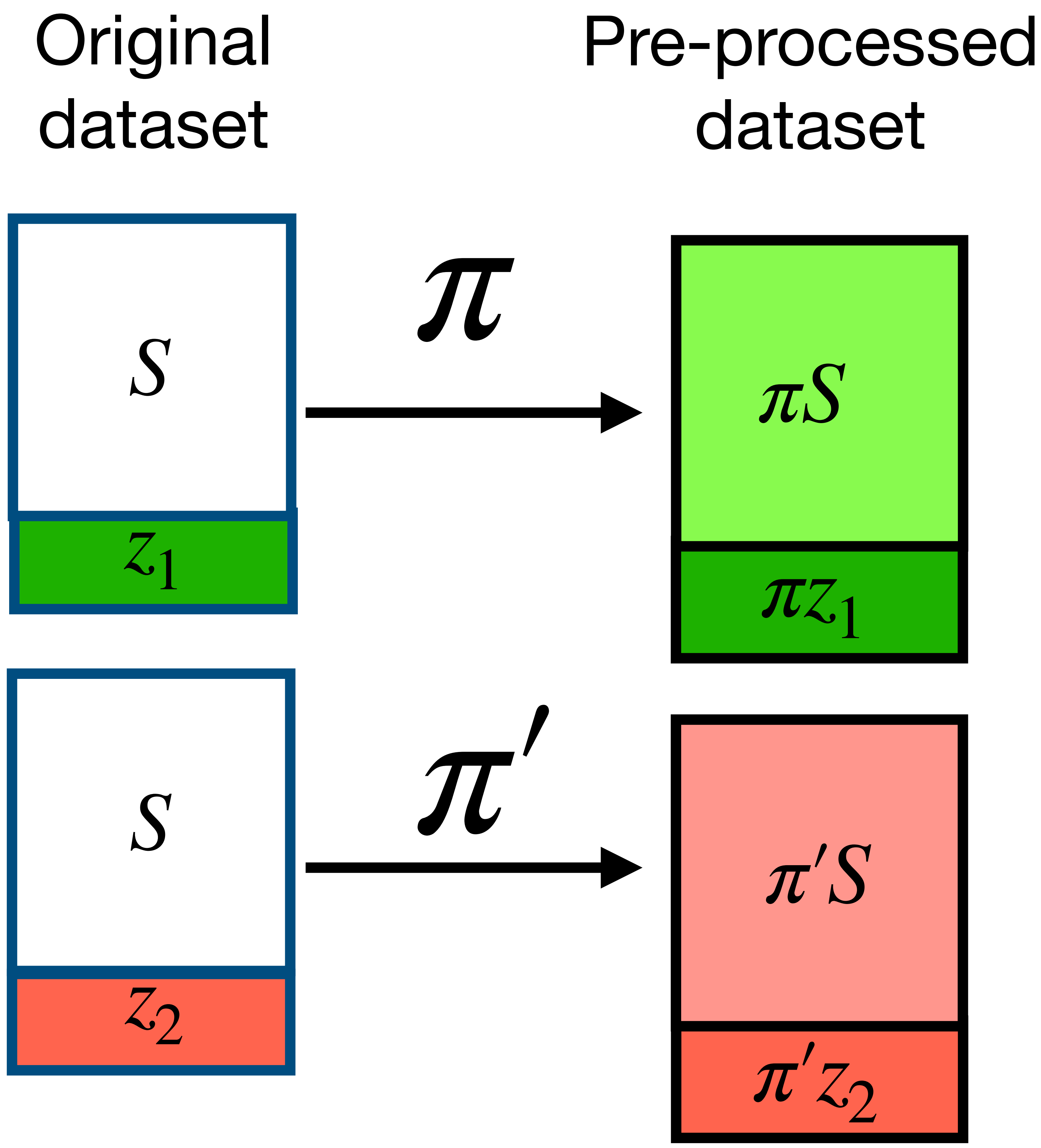

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"Privacy defined under a smooth distance metric gives more fine-grained analysis"

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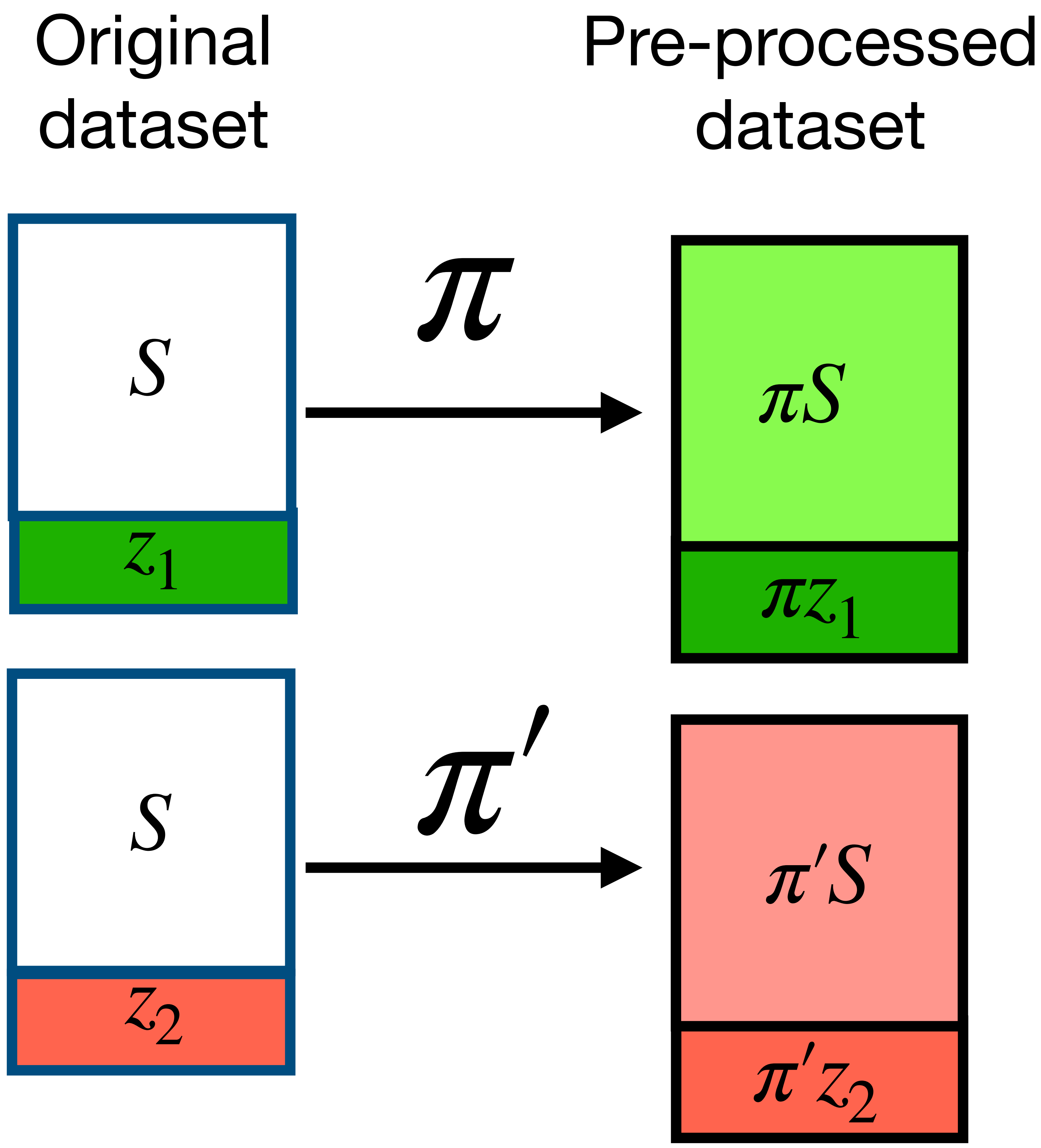

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“Privacy defined under a sr … rained analysis”

Smooth RDP

Renyi DP and Smooth Renyi DP (SRDP)

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For $\alpha > 1, m \in \mathbb{N}$, an algorithm $\mathscr{A}$ is $\varepsilon(\alpha)$-RDP if for any two datasets S and S' differing at m points- $d_H(S, S') \leq m$

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Stability against arbitrary perturbation on a fixed number of data points

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Stability against arbitrary perturbation on a fixed number of data points

For $\alpha > 1, \tau > 0$, an algorithm $\mathcal{A}$ is $\varepsilon(\alpha, \tau)$-SRDP if for any two datasets S and S' satisfying

$$\sum_{i=1}^{n} \|S_i - S_i'\|_2 \leq \tau$$

$$D_\alpha\left(\mathcal{A}(S) || \mathcal{A}(S')\right) \leq \varepsilon\left(\alpha, \tau\right)$$

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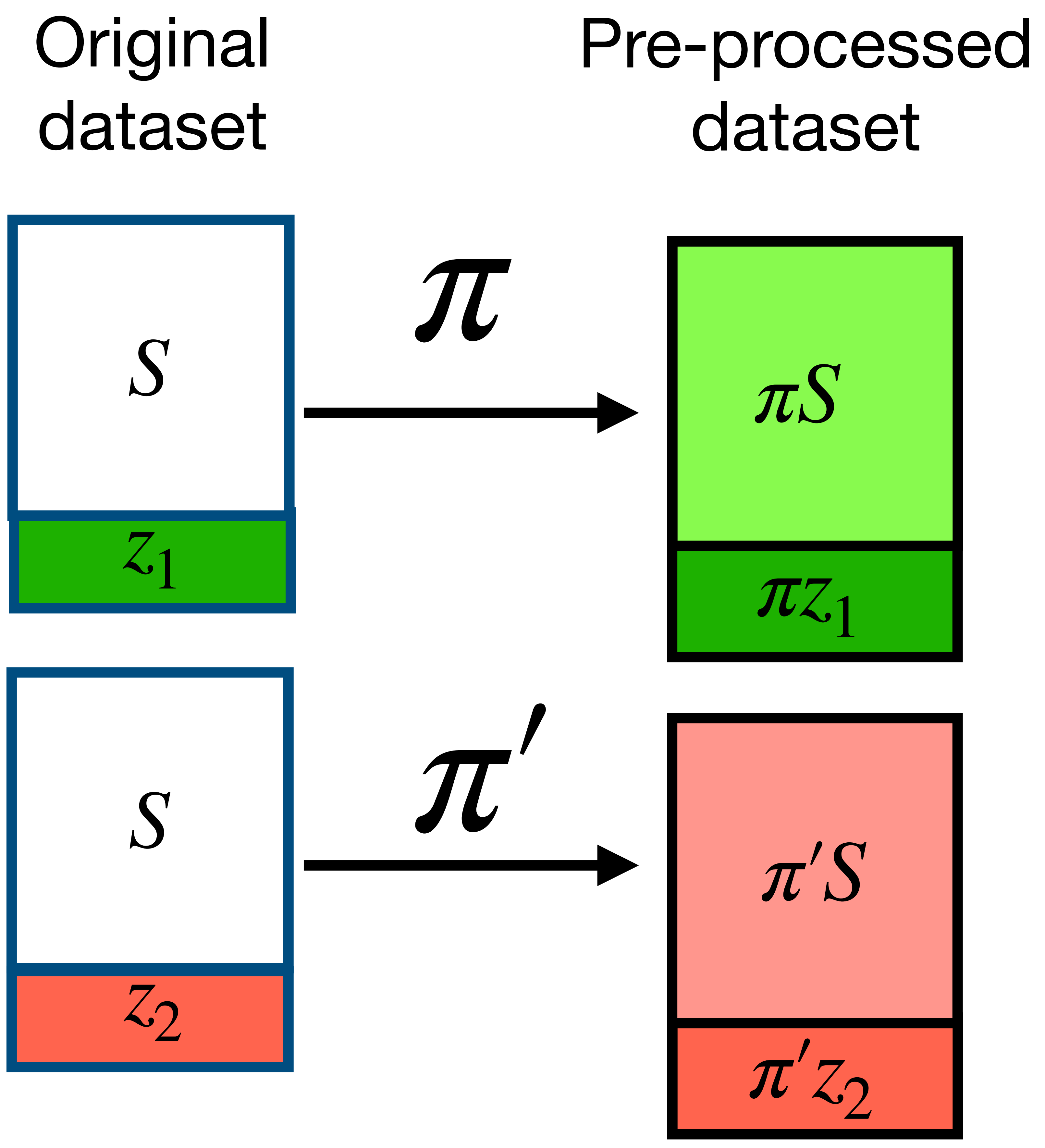
Stability against bounded perturbation on arbitrary number of data points

Most DP mechanisms satisfy Smooth-RDP

Notation	Meaning	Mechanism	Assumptions	RDP	SRDP
f	Output function	$\mathcal{M}_G$	f is L-Lipschitz	$\frac{\alpha\varepsilon^2}{2}$	$\frac{\alpha L^2\tau^2\varepsilon^2}{2\Delta_f^2}$
		$\mathcal{M}_L$	f is L-Lipschitz	ε	$\frac{L\tau}{\Delta_f}\varepsilon$
Q	Score function	$\mathcal{M}_E$	Q is L-Lipschitz	ε	$\frac{L\tau}{\Delta_Q}\varepsilon$
ℓ	Loss function	$\mathcal{A}_{\text{GD}}$	ℓ is L-Lipschitz and μ-smooth, $\sigma = \frac{L\sqrt{T}}{\varepsilon n}$	$2\alpha\varepsilon^2$	$\frac{\alpha\mu^2\tau^2\varepsilon^2}{2L^2}$
T	Number of iteration	$\mathcal{A}_{\text{SGD}-\text{samp}}$	ℓ is L-Lipschitz and μ-smooth, $\sigma = \Omega(L\sqrt{T}/\varepsilon n)$, inverse point-wise divergence γ, $1 \leq \alpha \leq \min\left\{\frac{\sqrt{T}}{\varepsilon}, \frac{L^2T}{\varepsilon^2 n^2}\log\frac{n^2\epsilon}{L\sqrt{T}}\right\}$	$\frac{\alpha^2\varepsilon^2}{2}$	$\frac{\alpha\mu^2\tau^2\varepsilon^2\gamma^2}{2L^2}$
η	Learning rate	$\mathcal{A}_{\text{SGD}-\text{iter}}$	ℓ is convex, L-Lipschitz and μ-smooth, $\sigma = \frac{8\sqrt{2\log n}\eta L}{\varepsilon\sqrt{n}}$, $\varepsilon = O(1/n\alpha^2)$, maximum divergence κ_τ, $L\sqrt{2\alpha(\alpha-1)} \leq \sigma$	$\frac{\alpha\varepsilon^2}{2}$	$\frac{\alpha\tau^2\mu^2 n\log(n-\kappa_\tau+2)}{2(n-\kappa_\tau+1)L^2\log n}$

Table 1: RDP and SRDP parameters of DP mechanisms.

Sensitivity of pre-processing algorithms



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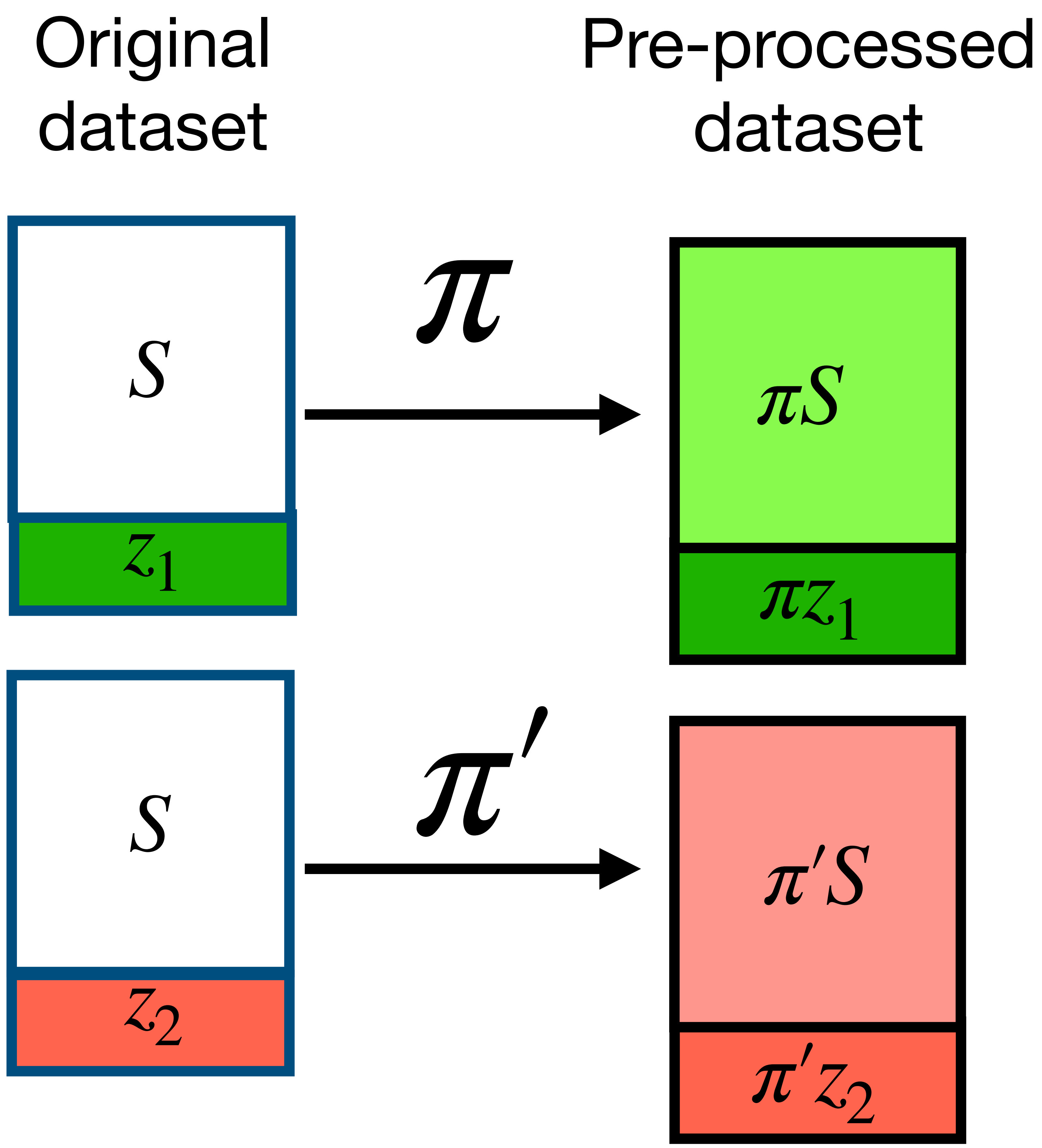

$$L_2 \text{ sensitivity} = \max_{x \in S} \max_{S,S'} \|\pi_S(x) - \pi_{S'}(x)\|_2$$

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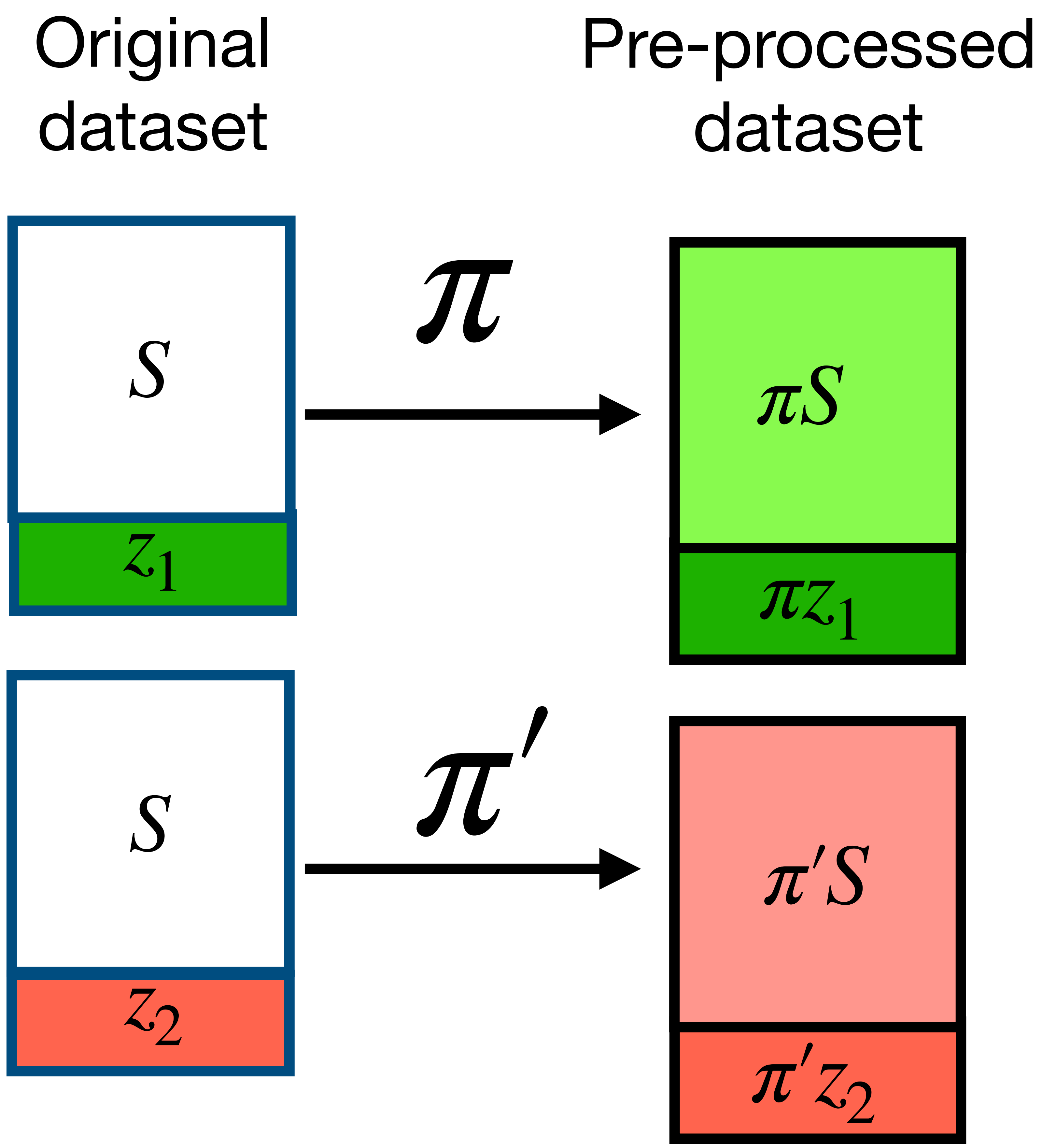

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$$L_\infty \text{ sensitivity: } \max_{x \in S} \max_{S,S'} \|\pi_S(x) - \pi_{S'}(x)\|_0$$

Main Result

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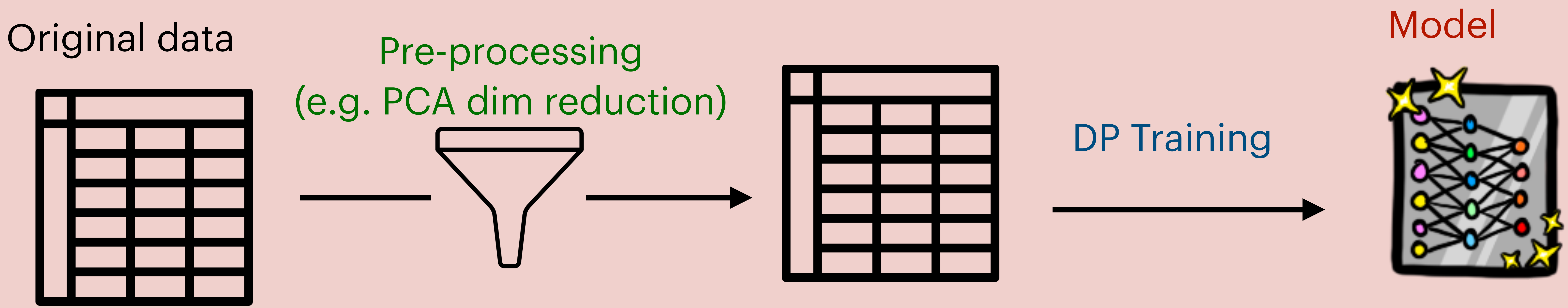
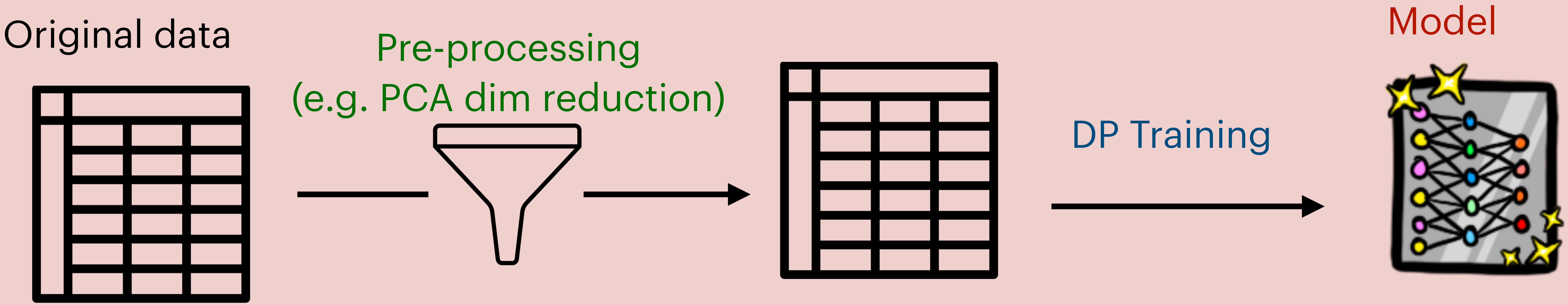

Original data
Pre-processing
(e.g. PCA dim reduction)
DP Training
Model

Main Result



Assumption 1:
The pre-processing algorithm has
L_2 sensitivity Δ_2 and
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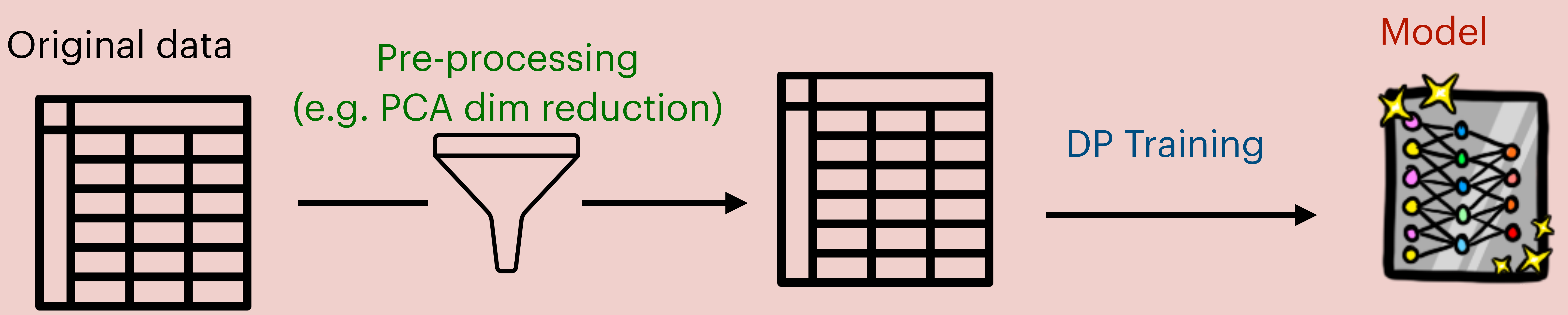

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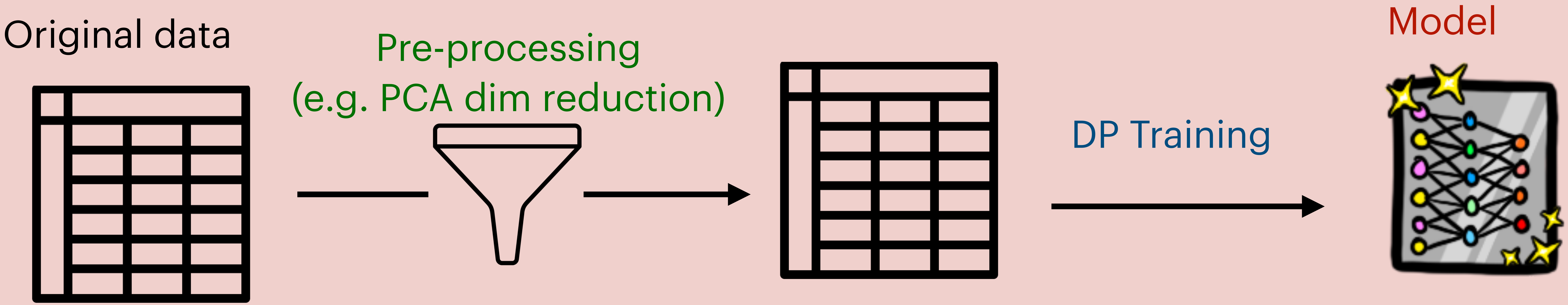

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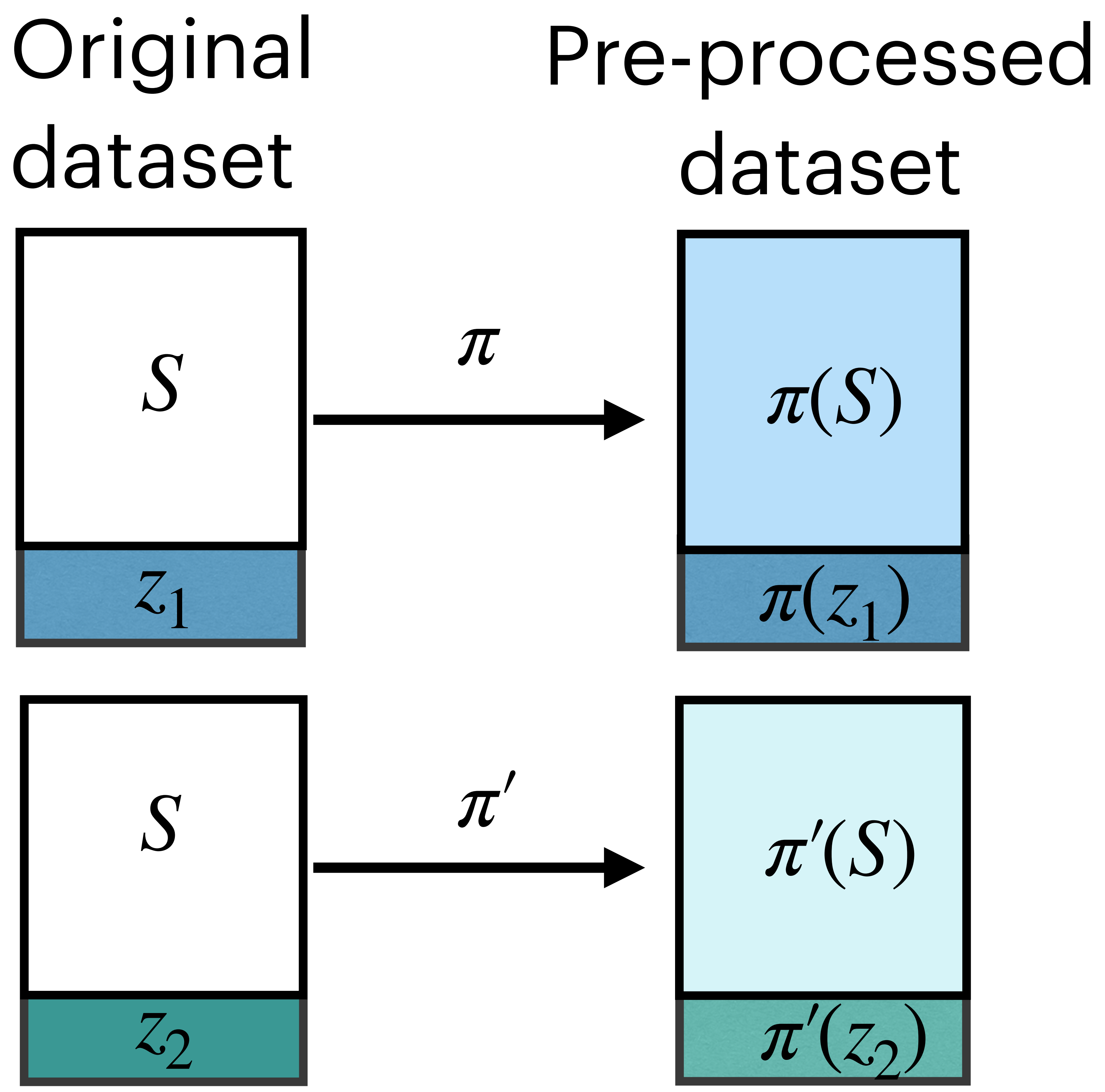
The pre-processed DP pipeline is $(\alpha, \hat{\varepsilon})$-RDP, where

$$\hat{\varepsilon} = \frac{2\alpha - 1}{2(\alpha - 1)}\left(\tilde{\varepsilon}(2\alpha, \Delta_2\Delta_\infty) + \varepsilon(2\alpha)\right).$$

Proof Sketch



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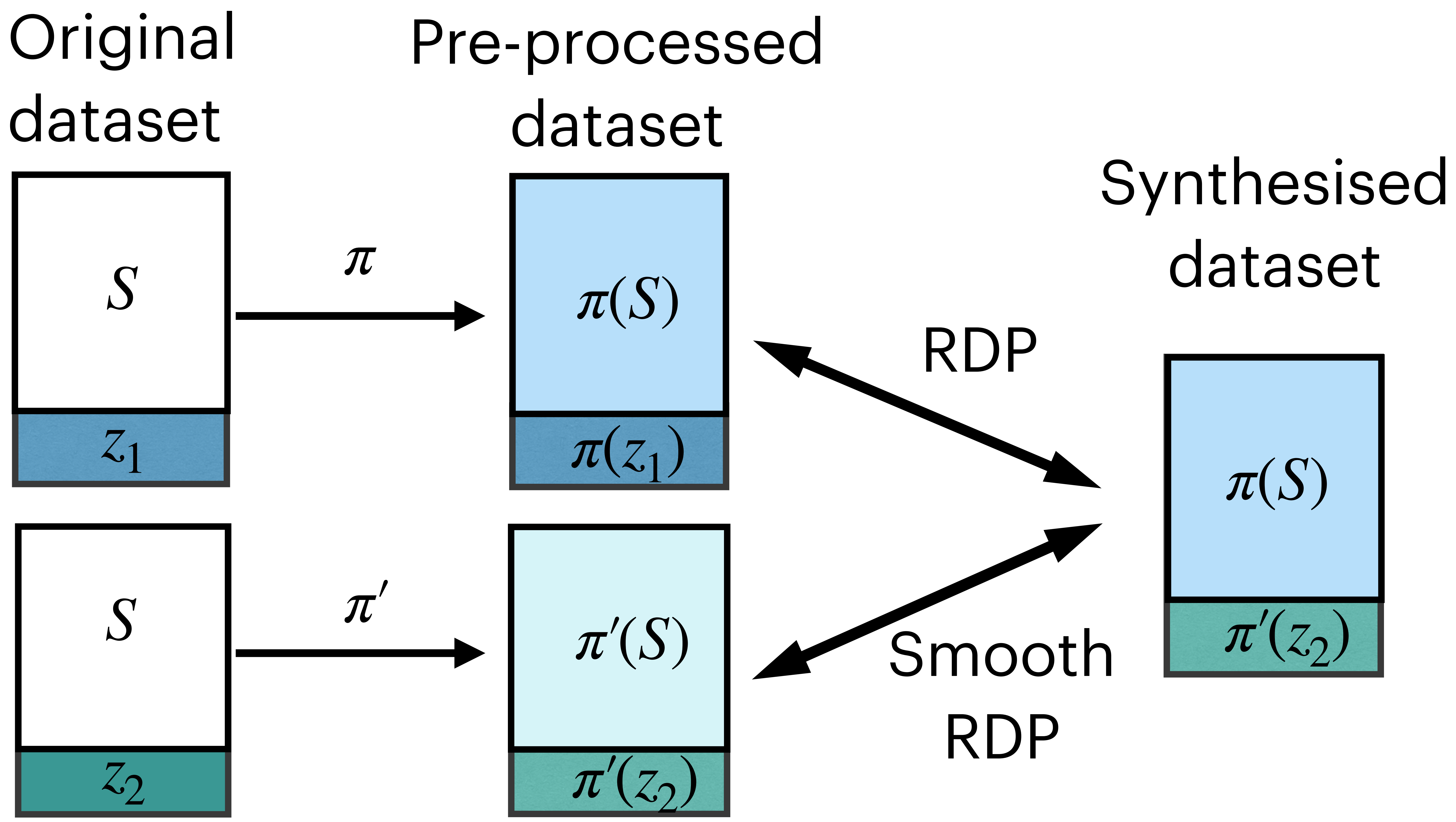
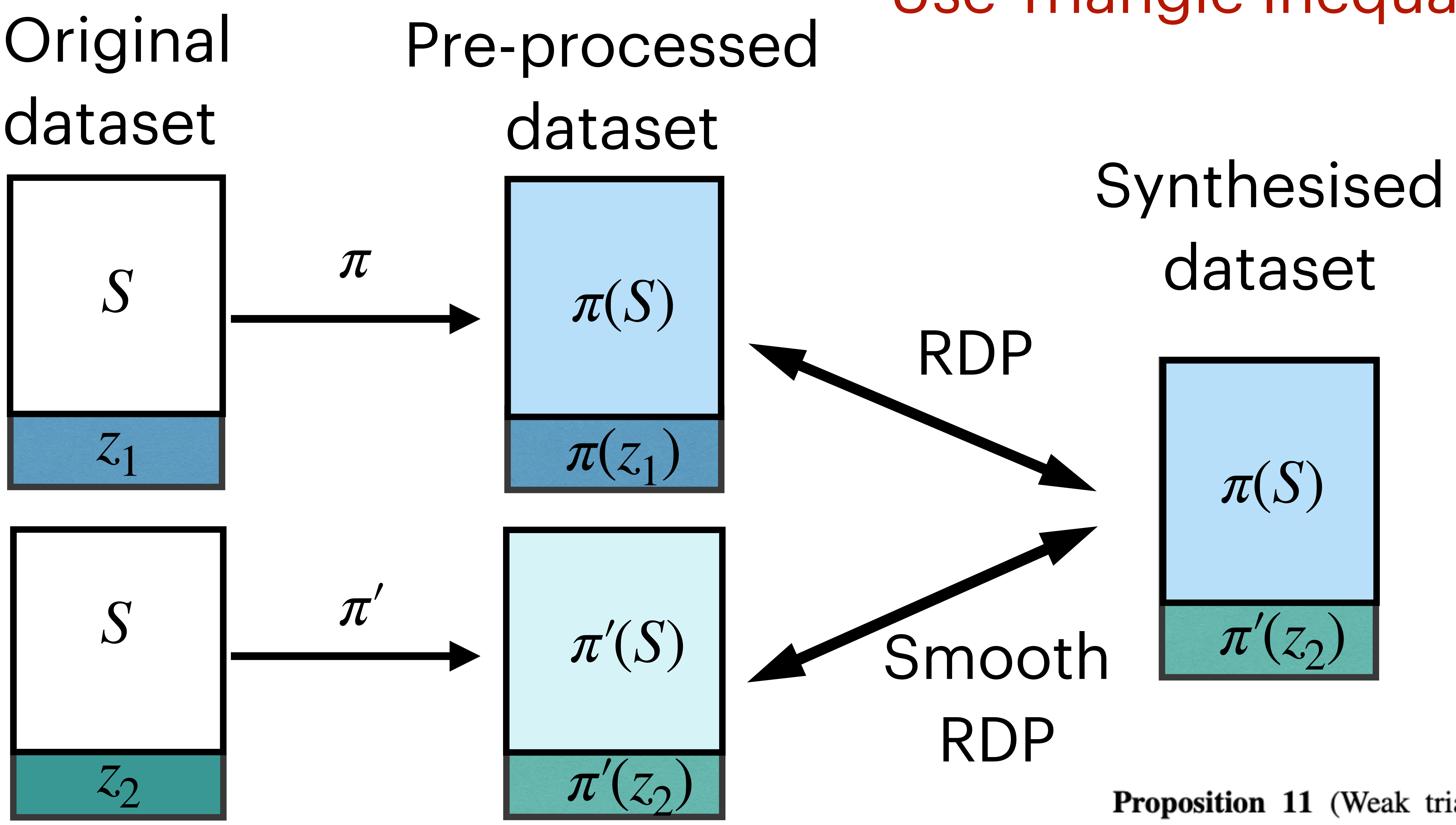


Proof Sketch

Use Triangle Inequality for Renyi DP



Proposition 11 (Weak triangle inequality). *Let P, Q, R be distributions on $\mathcal{R}$. Then for $\alpha > 1$ and for any $p, q > 1$ satisfying $1/p + 1/q = 1$ it holds that*

$$D_\alpha(P\|Q) \le \frac{\alpha - 1/p}{\alpha - 1} D_{p\alpha}(P\|R) + D_{q(\alpha-1/p)}(R\|Q).$$

Visualising the advantage

Visualising the advantage

	Quantization	Mean imputation	PCA	Standard Scaling
$\mathcal{M}_G$	$1.05\alpha\varepsilon^2\left(1+\eta^2p^2\right)$	$1.05\alpha\varepsilon^2\left(1+\frac{4p^2}{(n-p)^2}\right)$	$1.05\alpha\varepsilon^2\left(1+\frac{12.2^2}{(\delta^k_{\min})^2}\right)$	$1.05\alpha\varepsilon^2\left(1+\frac{4}{\sigma^3_{\min}}\right)$
$\mathcal{A}_{\mathrm{GD}}$	$1.05\alpha\varepsilon^2\left(4+\eta^2p^2\right)$	$4.2\alpha\varepsilon^2\left(1+\frac{p^2}{(n-p)^2}\right)$	$1.05\alpha\varepsilon^2\left(4+\frac{12.2^2}{(\delta^k_{\min})^2}\right)$	$4.2\alpha\varepsilon^2\left(1+\frac{1}{\sigma^3_{\min}}\right)$
$\mathcal{M}_L/\mathcal{M}_E$	$\varepsilon\left(1+\eta p\right)$	$\varepsilon\left(1+\frac{2p}{n-p}\right)$	$\varepsilon\left(1+\frac{12.2}{\delta^k_{\min}}\right)$	$\varepsilon\left(1+\frac{4}{\sigma^3_{\min}}\right)$
$\mathcal{A}_{\mathrm{SGD-samp}}$	–	–	$1.05\alpha\varepsilon^2\left(2\alpha+\frac{12.2^2}{(\delta^k_{\min})^2}\right)$	$2.1\alpha\varepsilon^2\left(\alpha+\frac{8}{\sigma^6_{\min}}\right)$
$\mathcal{A}_{\mathrm{SGD-iter}}$	$1.1\alpha\varepsilon^2\left(1+\frac{\eta^2p^2\frac{n}{\log n}}{\frac{n-p}{\log n-p}}\right)$	$1.1\alpha\varepsilon^2\left(1+\frac{4p^2\frac{n}{\log n}}{\frac{(n-p)^3}{\log n-p}}\right)$	–	–

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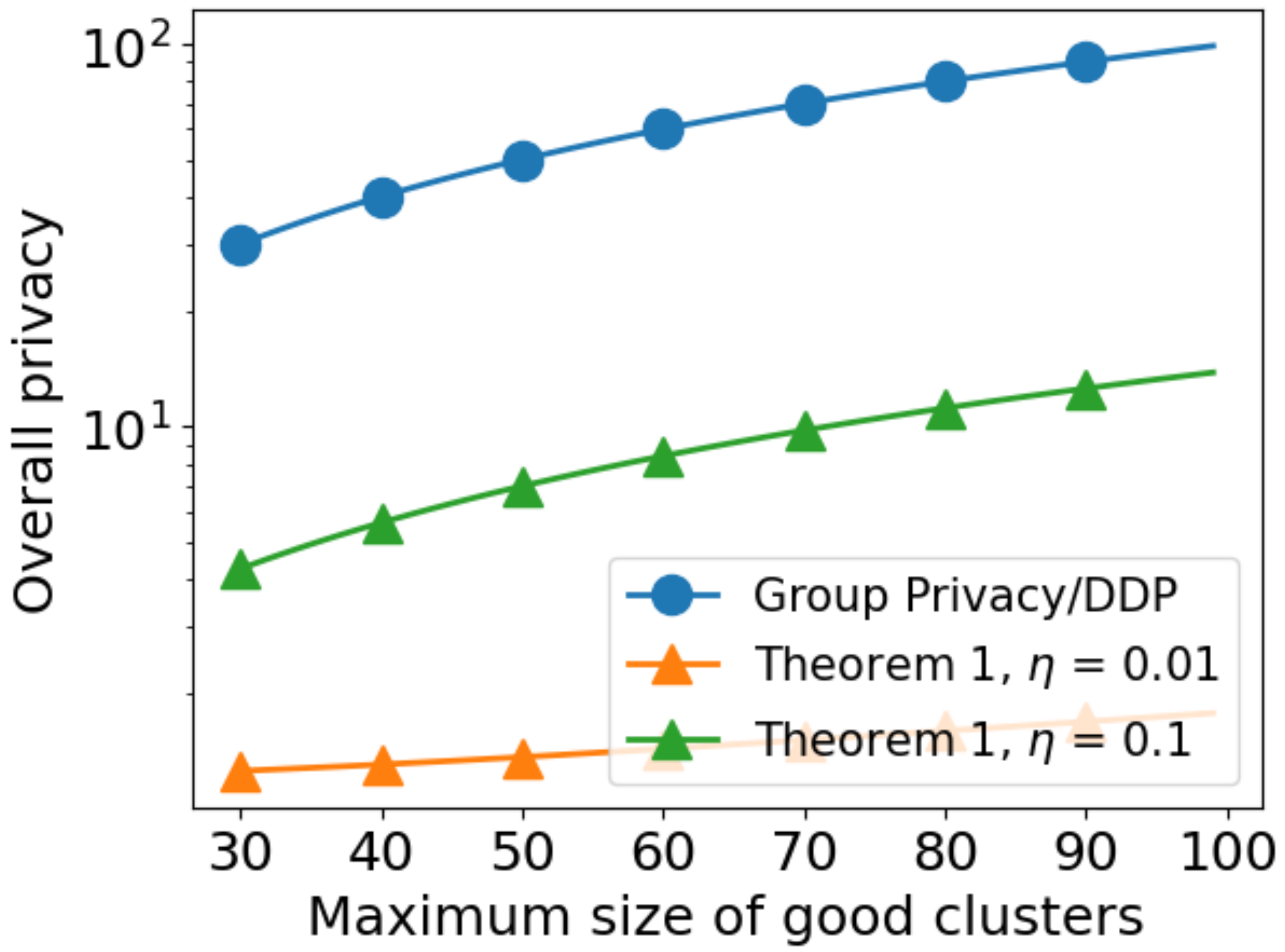
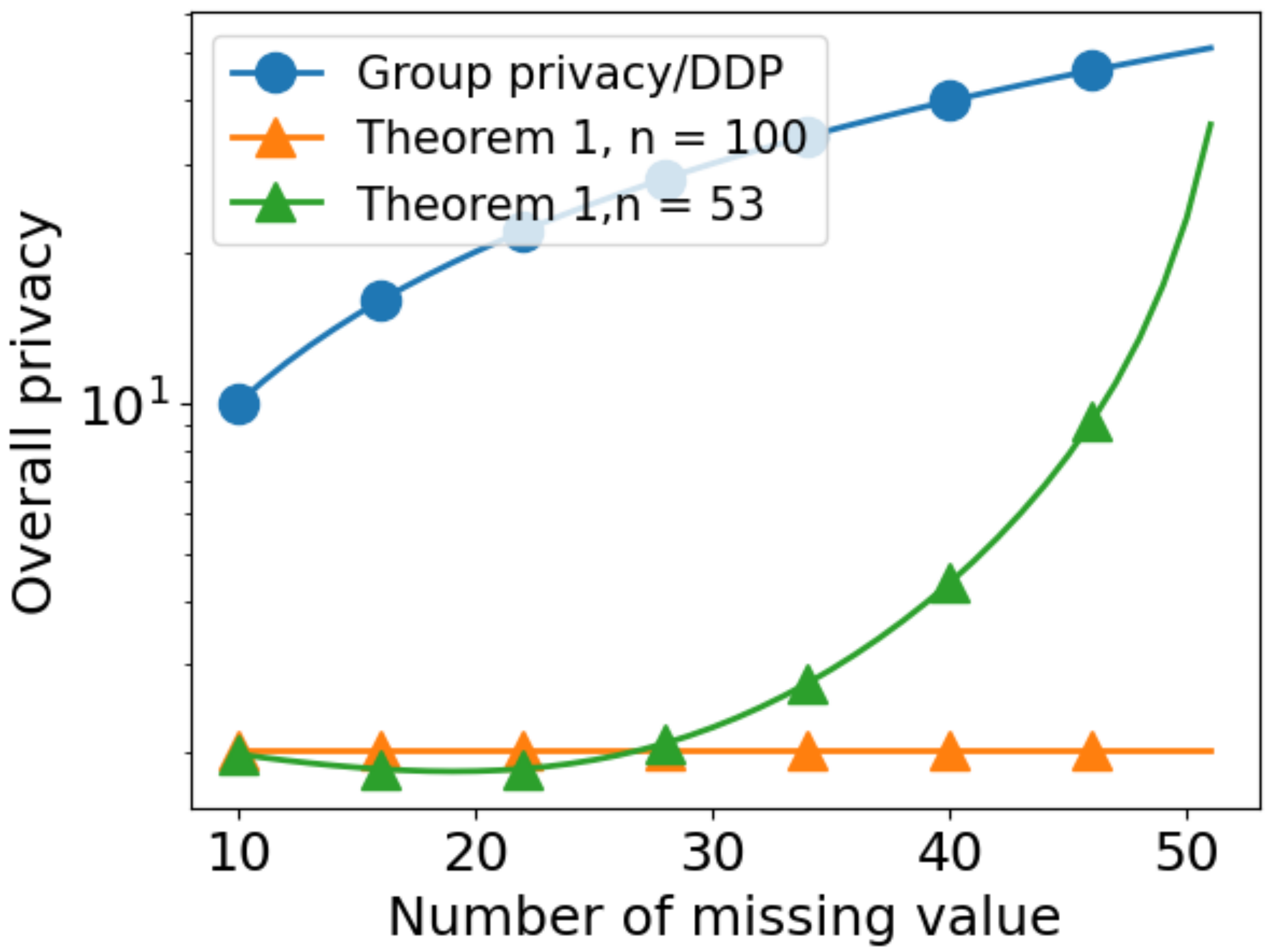
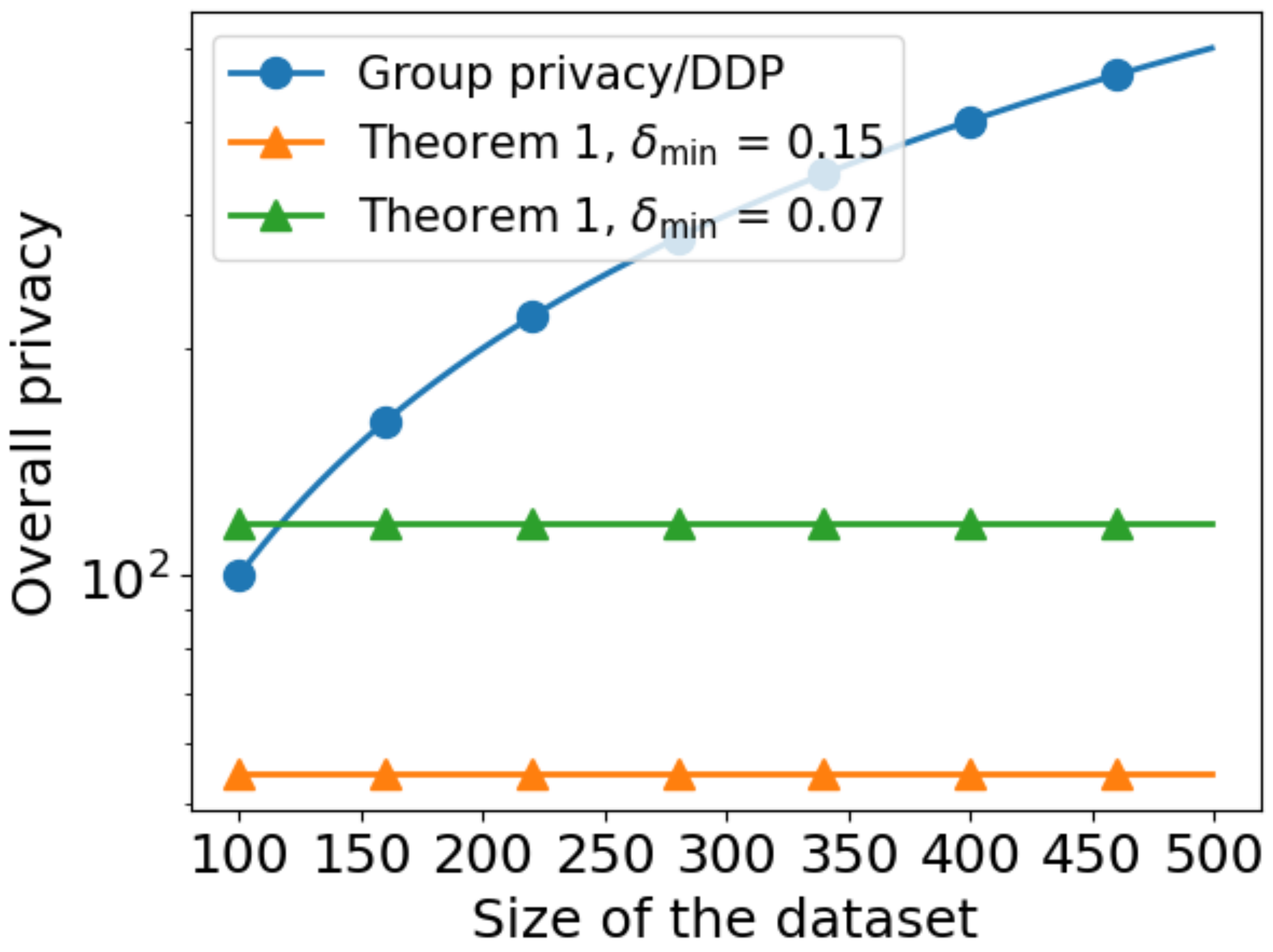


Quantization



Mean imputation



PCA

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DP Online Learning

Price of Privacy

Price of Privacy

Bounds for #mistakes

Price of Privacy

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	Offline Learning	Online Learning
Non-private algorithm		
Private algorithm		

Price of Privacy

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Private algorithm	$\leq \text{Ldim}^6$ [Ghazi et al., 2021]	

Price of Privacy

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Private algorithm	$\leq \text{Ldim}^6$ [Ghazi et al., 2021]	$\leq O\left(2^{2^{\text{Ldim}}} \frac{1}{\varepsilon} \log T/\delta\right)$ [Golowich & Livni, 2021]

[3] Golowich, N., & Livni, R. (2021). 1&. *Advances in Neural Information Processing Systems*

Price of Privacy

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Recall that, for **non-private** online learning, there is no dependency on T

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Q: Does the expected #mistakes inherently increase with T for private online learning?

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	Value at 0	Value at 1
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Simple online game

- Learner $\mathcal{A}$ guesses between $f^{(0)}$ and $f^{(1)}$

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- Learner $\mathcal{A}$ guesses between $f^{(0)}$ and $f^{(1)}$
- Adversary chooses $x \in \{0, 1\}$ s.t. $f^* = f^{(x)}$

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Simple online game

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$$F = \left\{ f^{(0)}, f^{(1)} \right\}$$

- Learner $\mathcal{A}$ guesses between $f^{(0)}$ and $f^{(1)}$
- Adversary chooses $x \in \{0, 1\}$ s.t. $f^* = f^{(x)}$
- Adversary can only give $(0, 0)$ and $(1, x)$

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Simple online game

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$$F = \left\{f^{(0)}, f^{(1)}\right\}$$

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$\exists \mathcal{A}$ s.t. $M = 1$

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But this completely leaks the privacy of all points in S.

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An algorithm $\mathbb{A}$ is β-*concentrated*, if $\exists x \in \{0,1\}$, such that if $\mathbb{P}(\mathbb{A}((\perp, \perp, \ldots, \perp)) = (x, x, \ldots, x)) \geq 1 - \beta$

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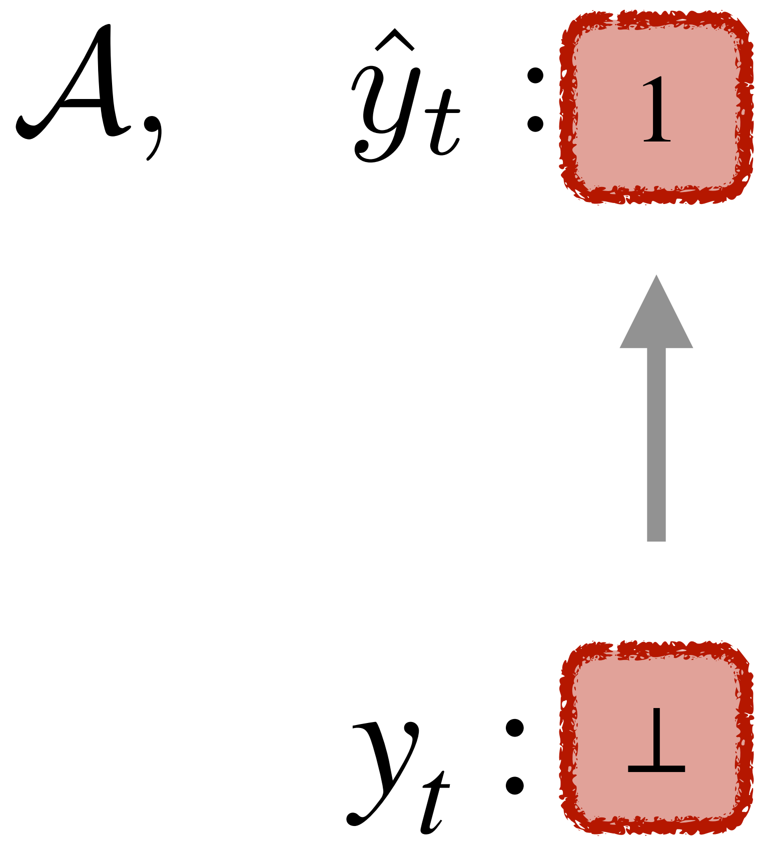

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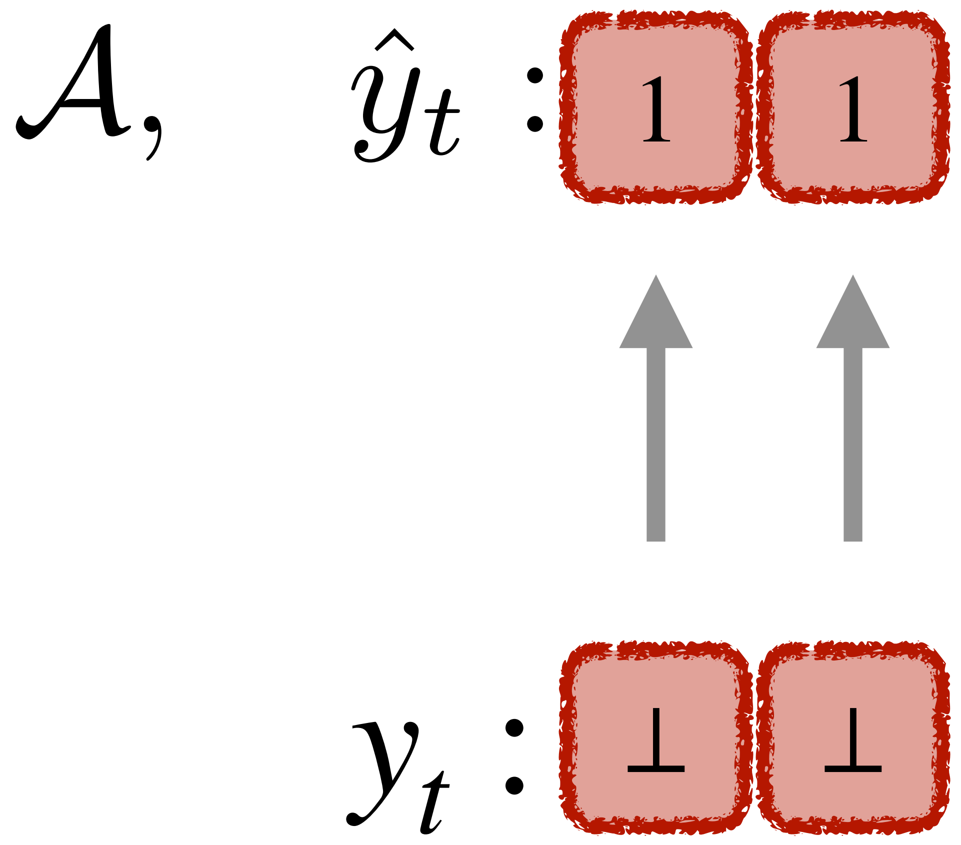

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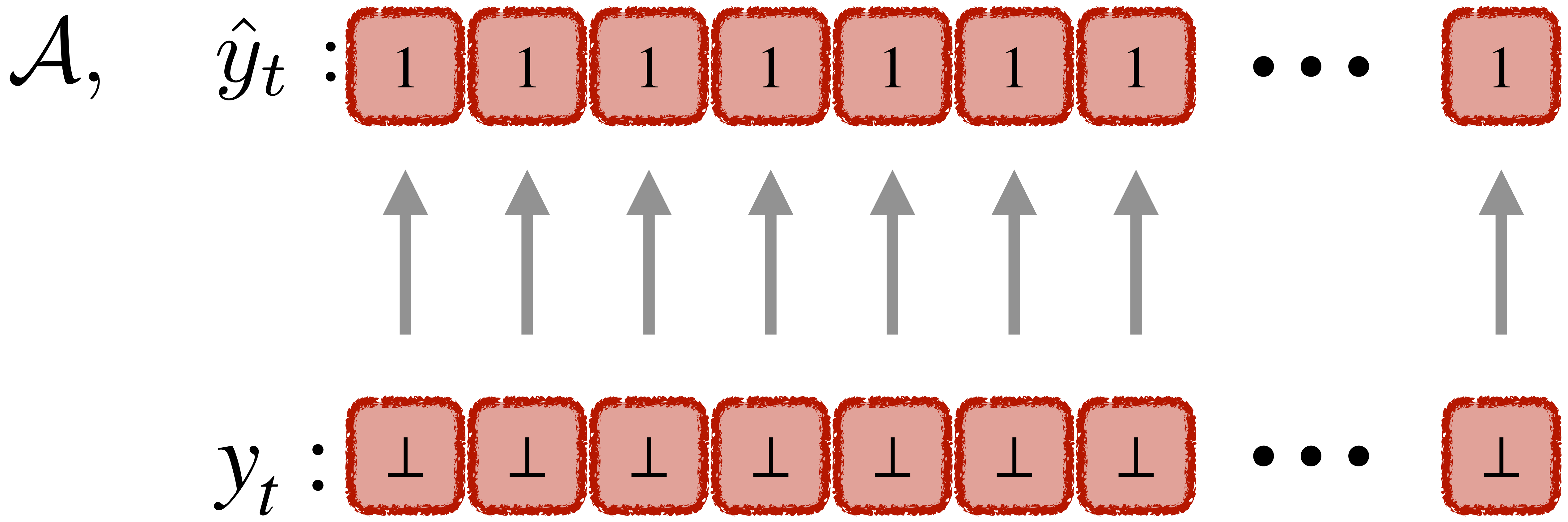

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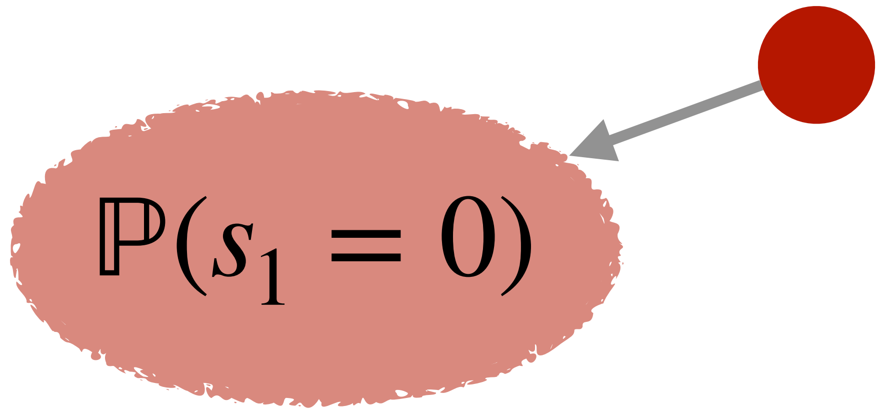

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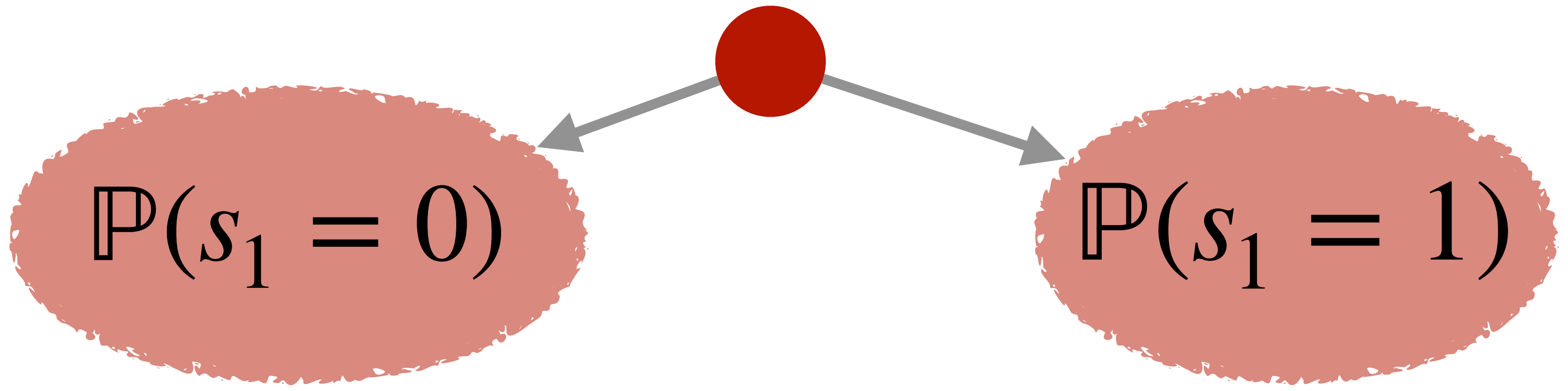

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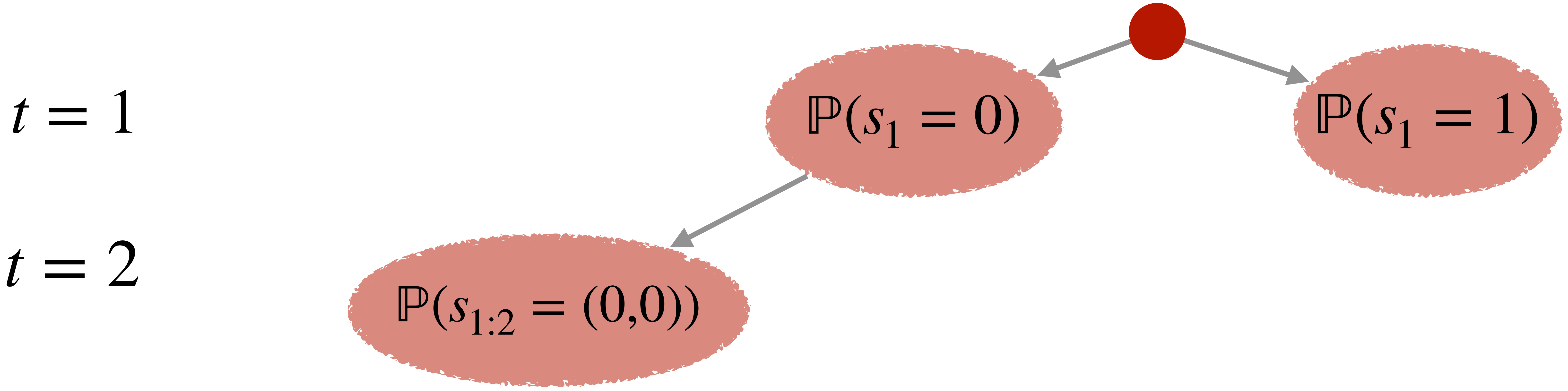

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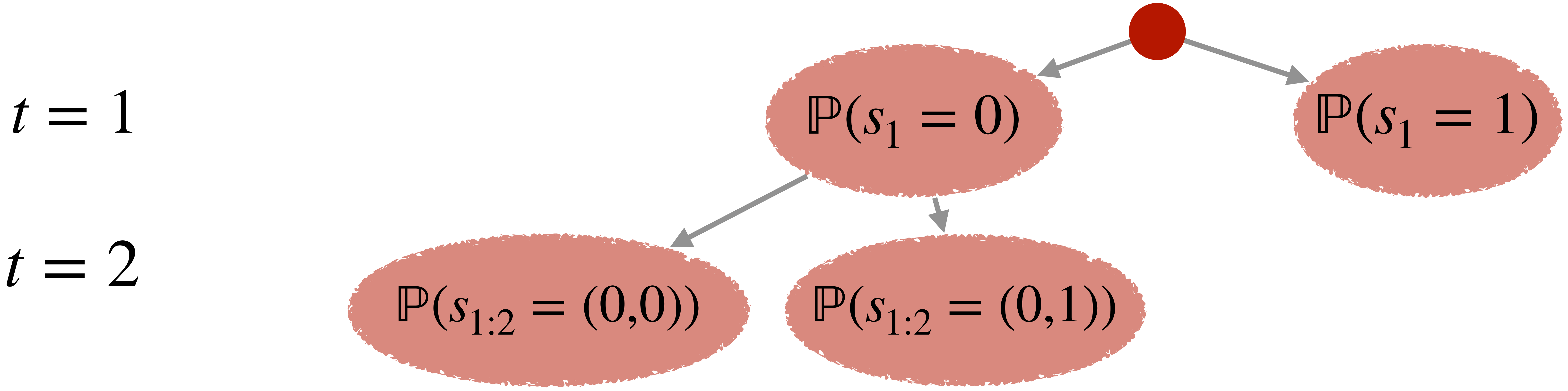

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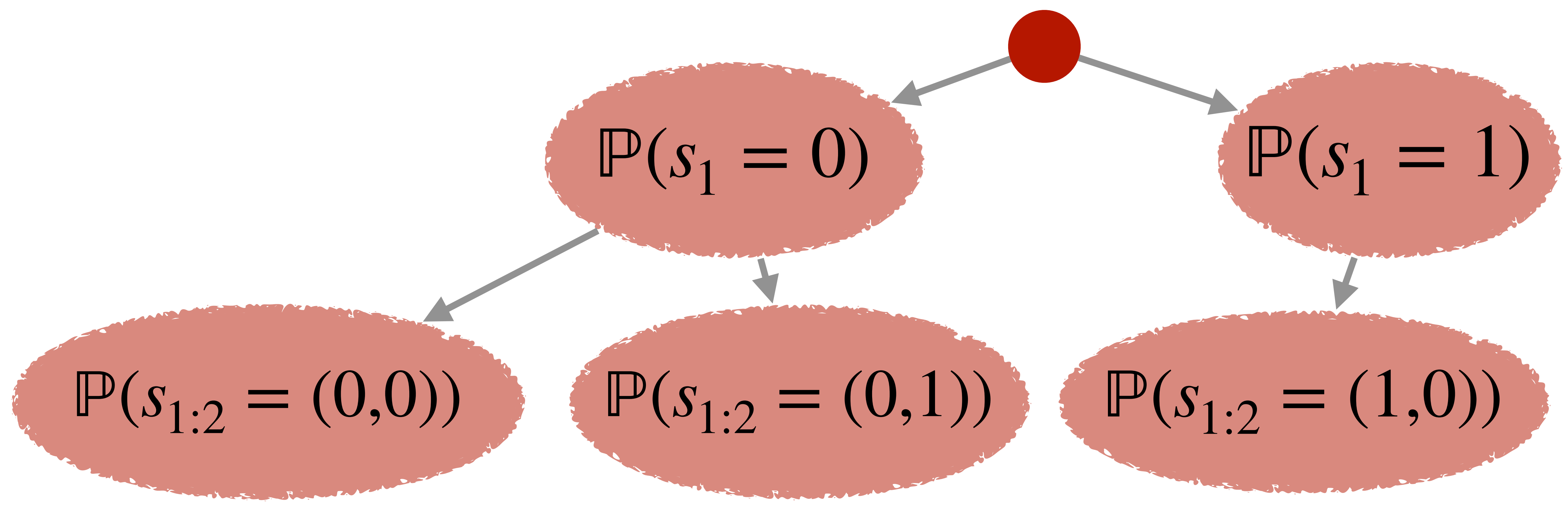

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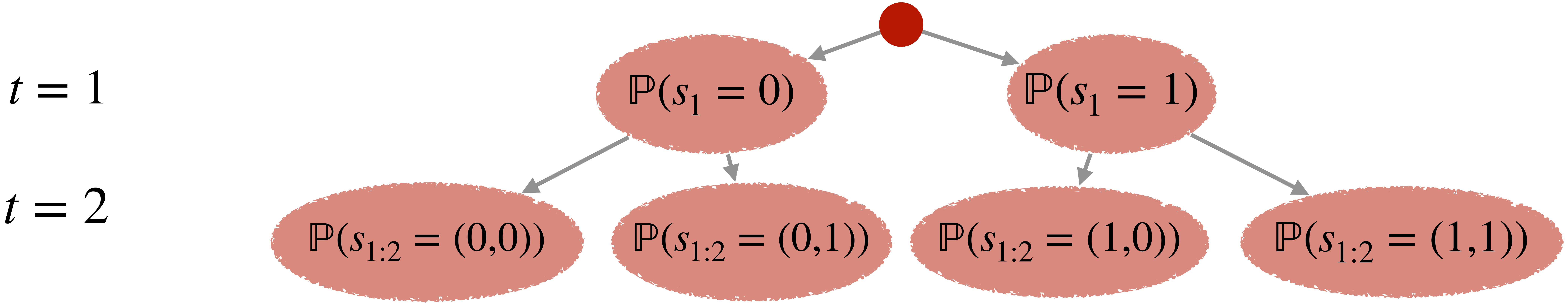

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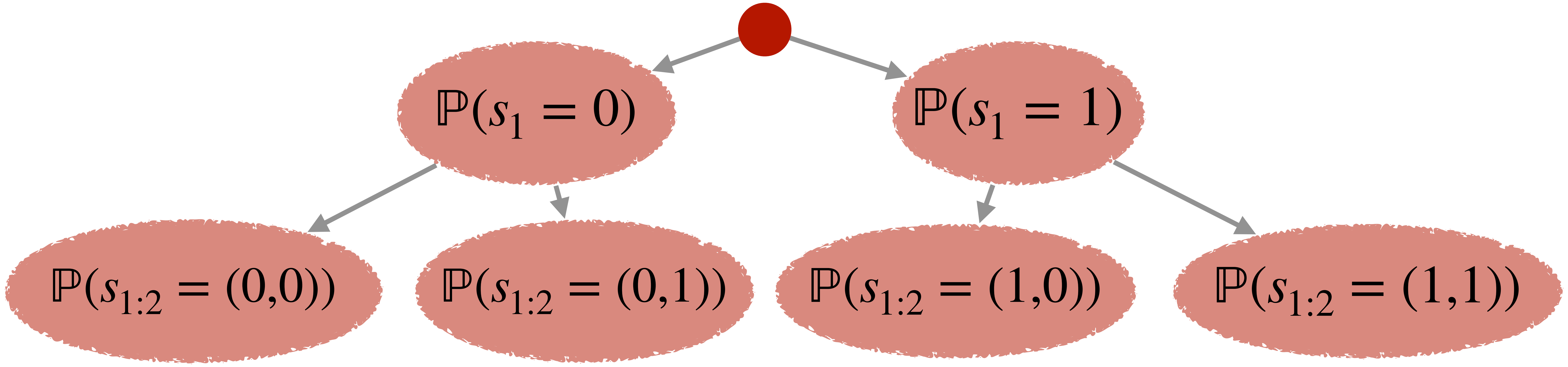

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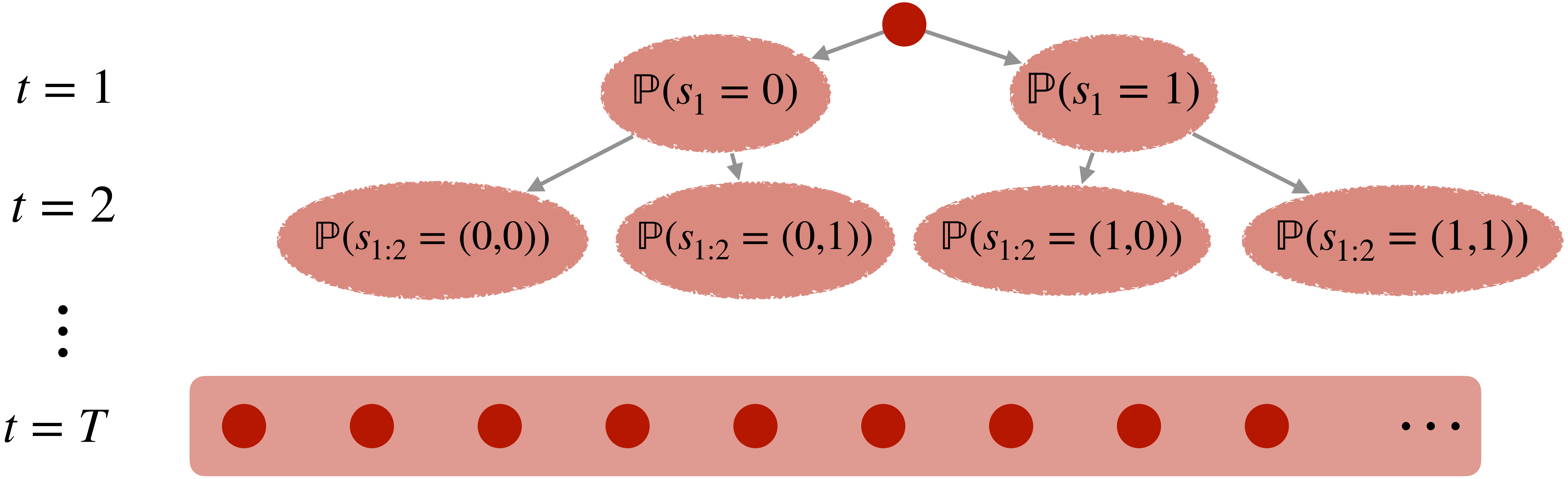

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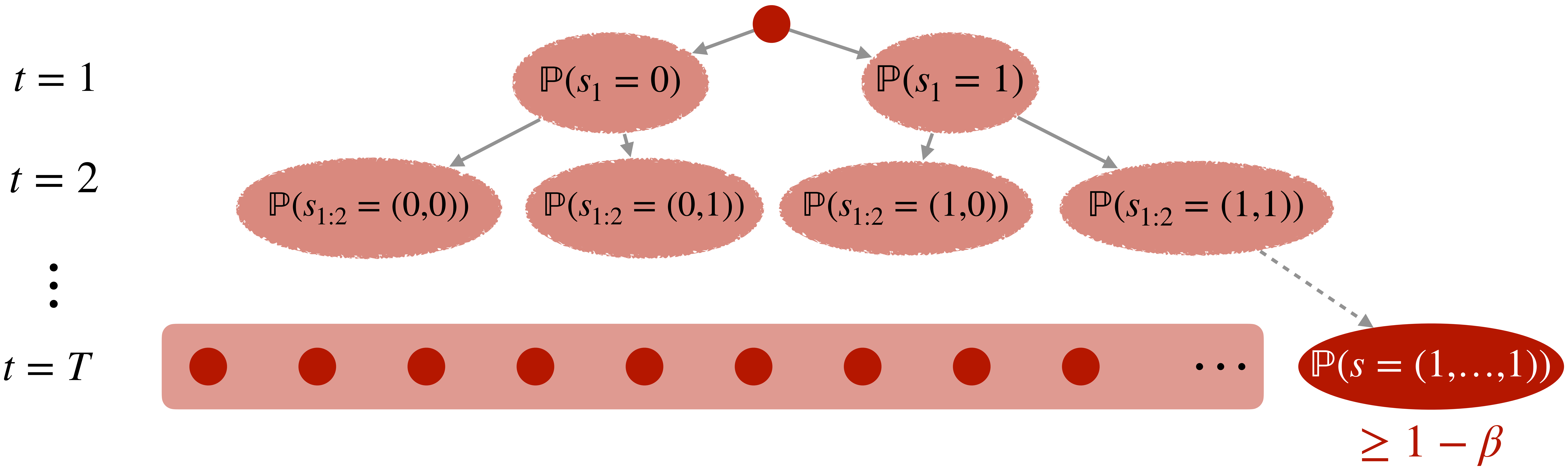

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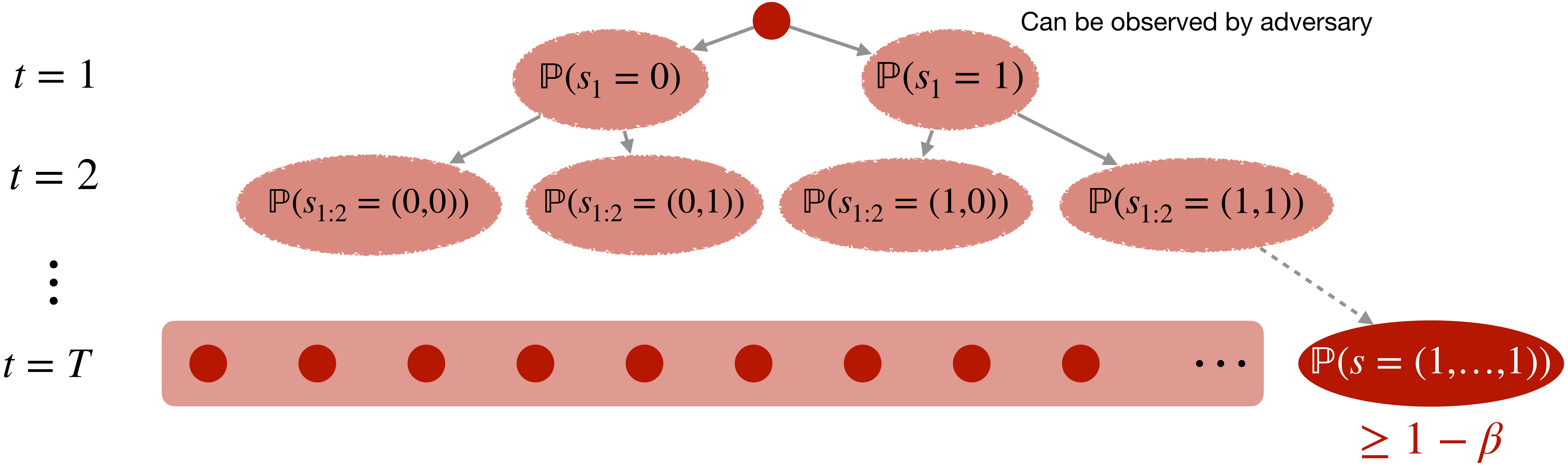

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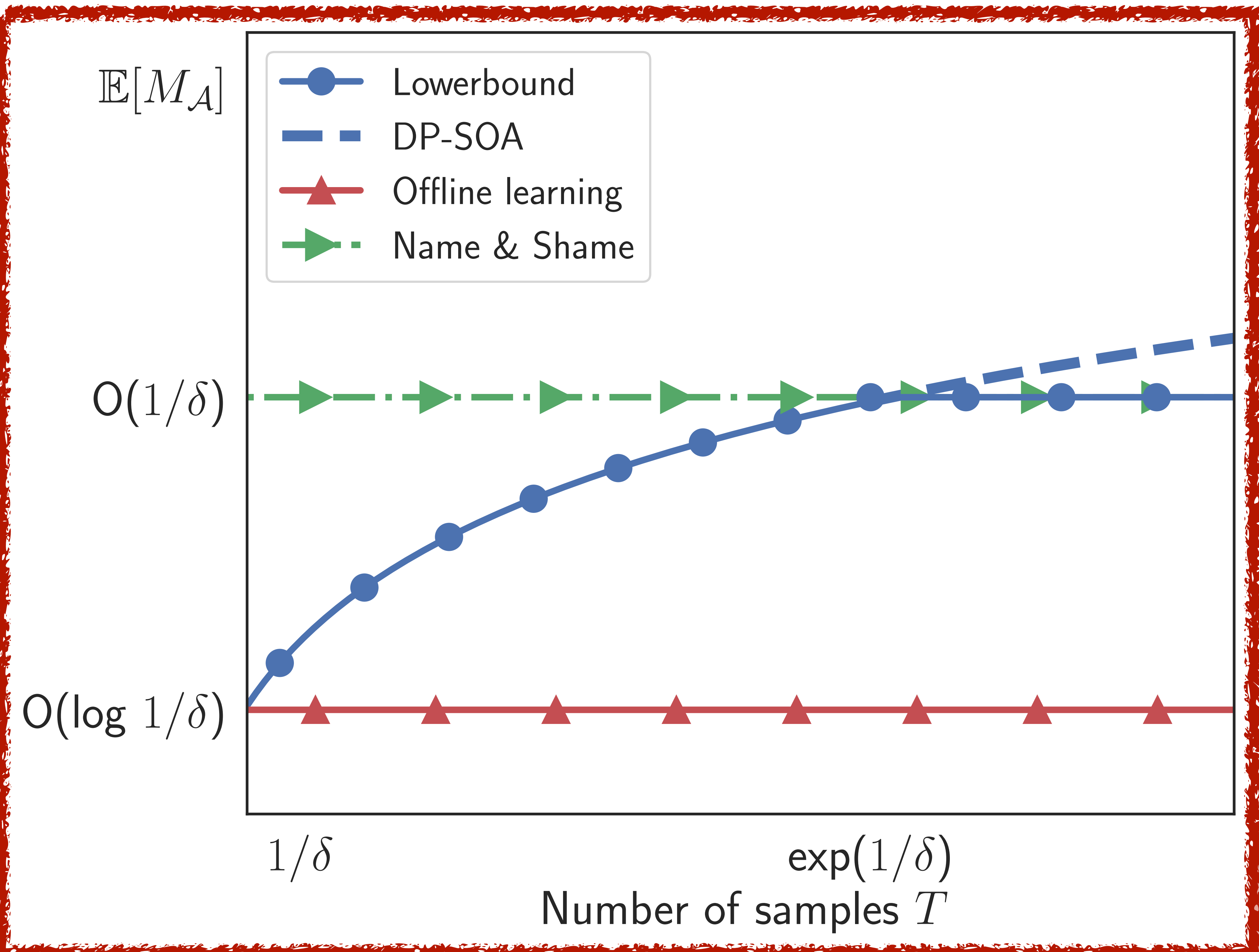

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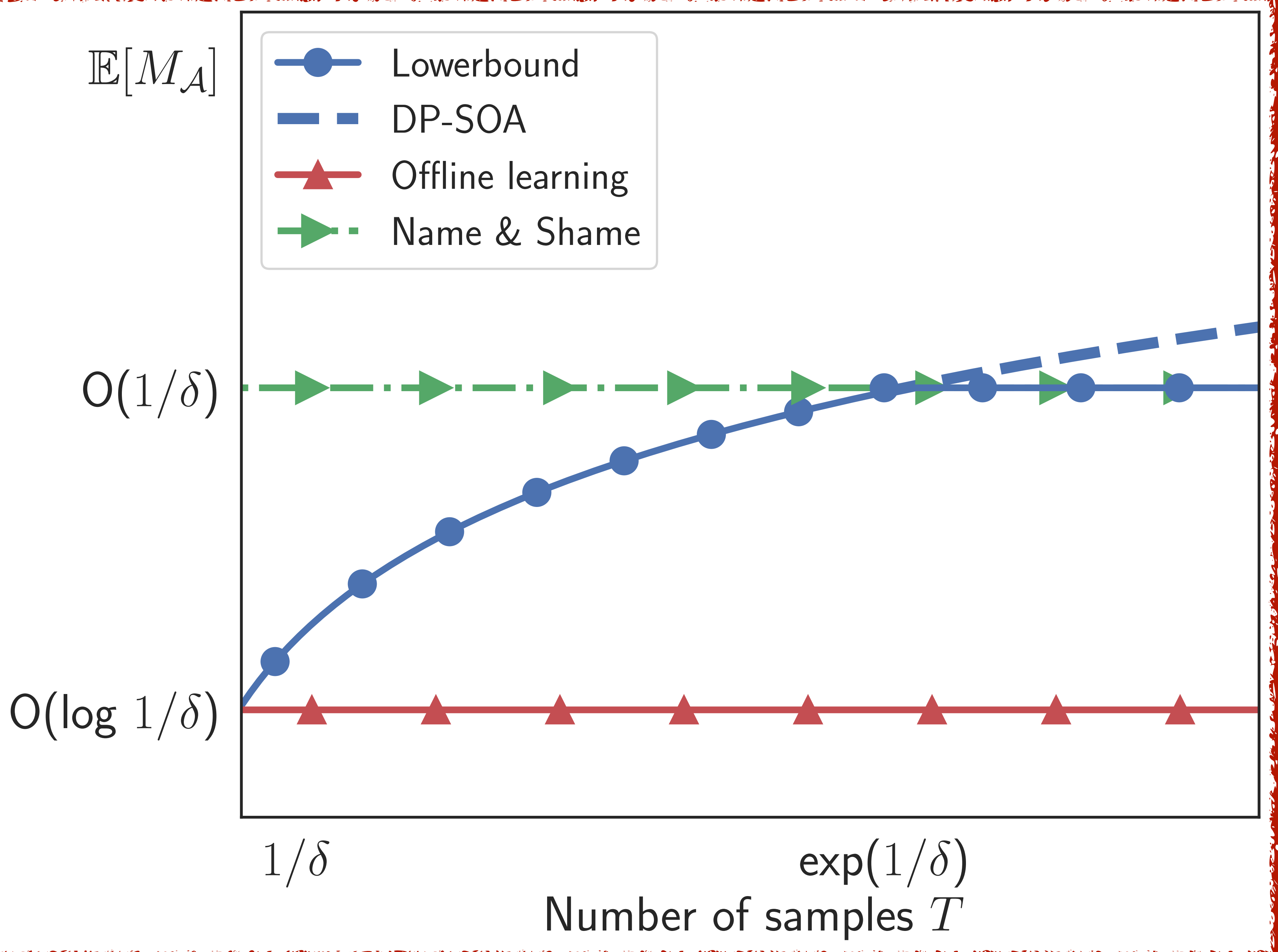
Proof technique:
Truly online lower bound



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Corollary

There exists a function class, Point_N, such that any (ε, δ)-private proper online algorithm must have $\mathbb{E}\left[\mathrm{M}\right] \geq \min\left(\frac{1}{\delta}, \frac{1}{1000\varepsilon}\log\frac{T}{\delta}\right)$ in the worst case.

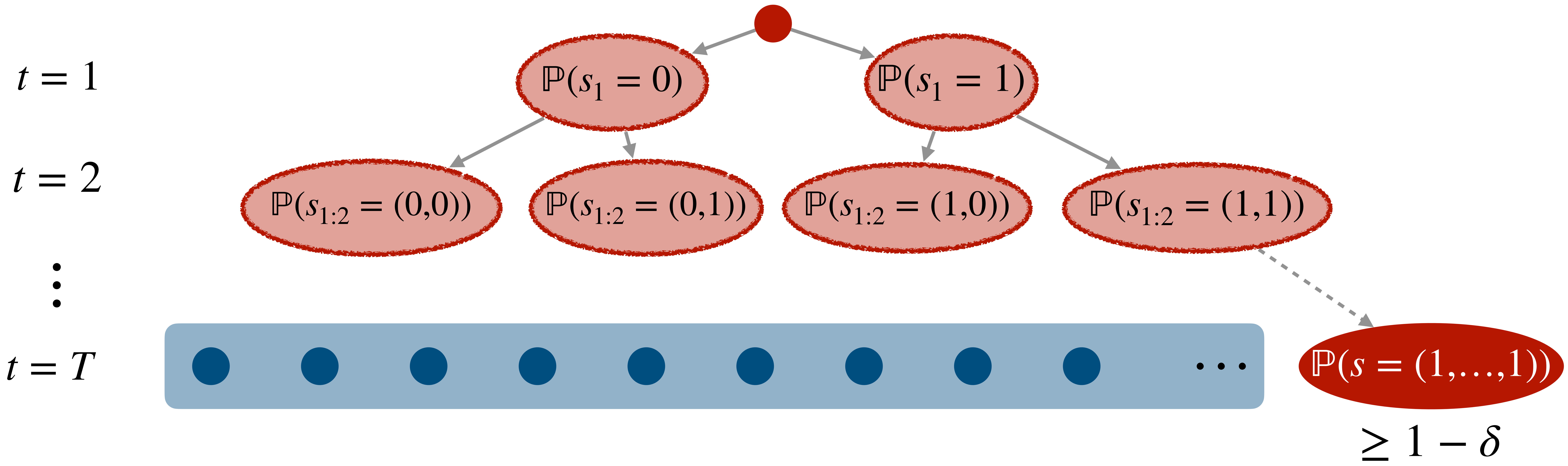
Concurrent work [CLNSS24]

For $N \geq \sqrt{T}$, Point_∞, any (ε, δ)-private online algorithm must have $\mathbb{E}\left[\mathrm{M}\right] \geq \min\left(\frac{1}{\delta}, \frac{1}{1000\varepsilon}\log T\right)$ in the worst case.

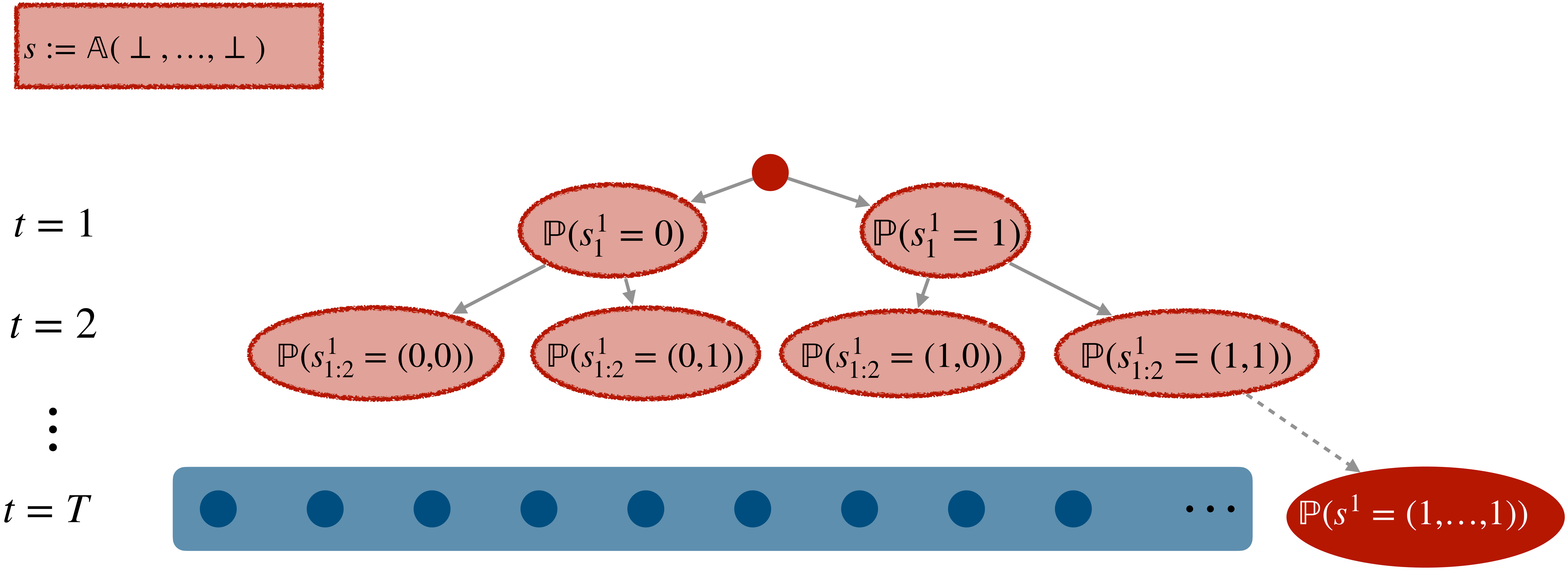
Proof Sketch

Concentration

$s := \mathbb{A}(\perp, \ldots, \perp)$



Applying DP on the `correct' event



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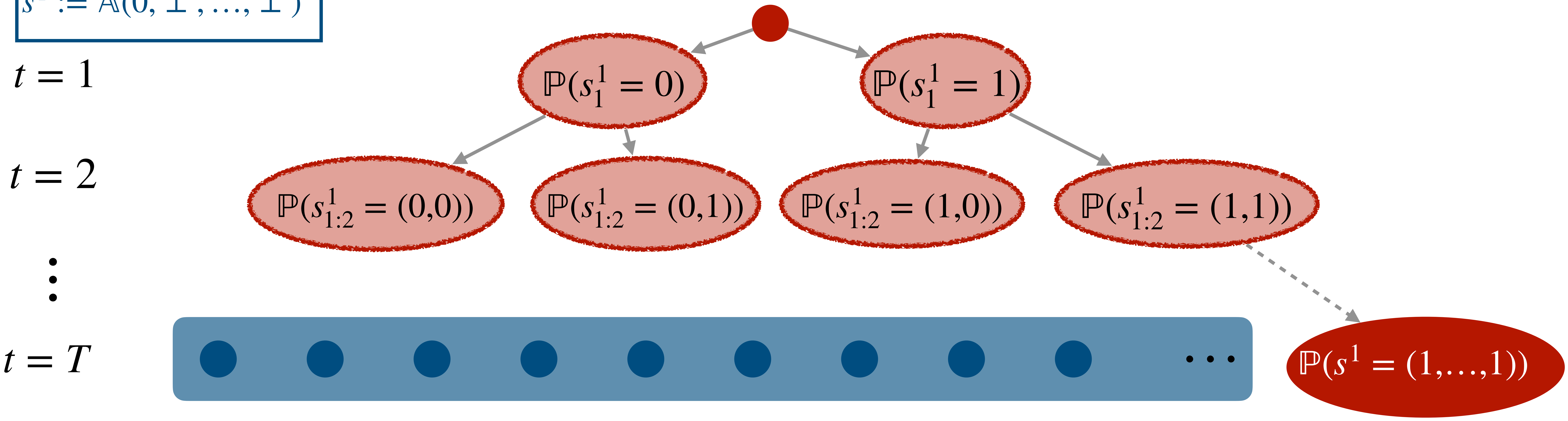

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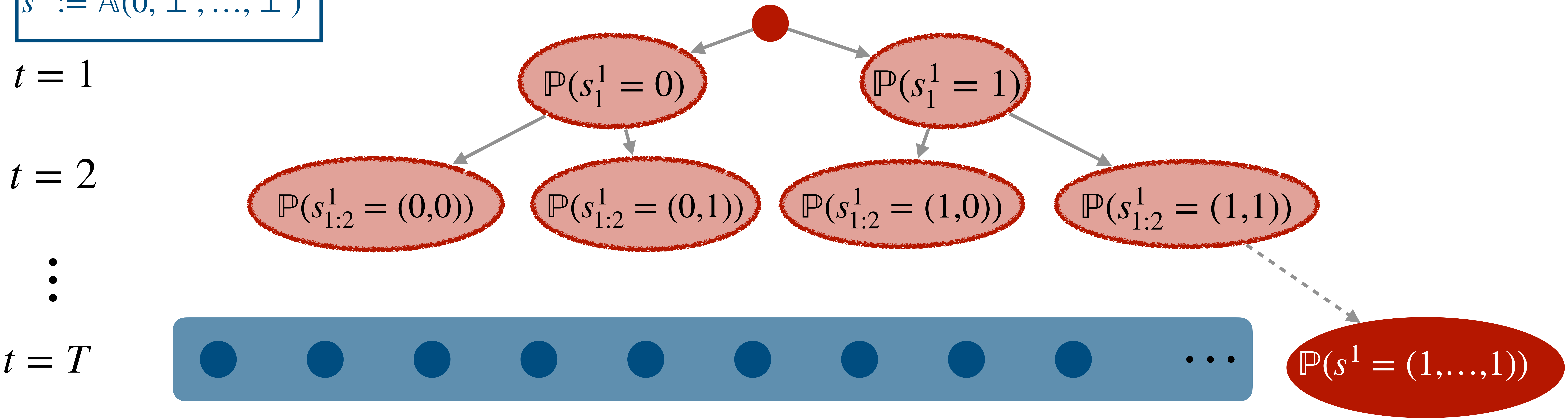

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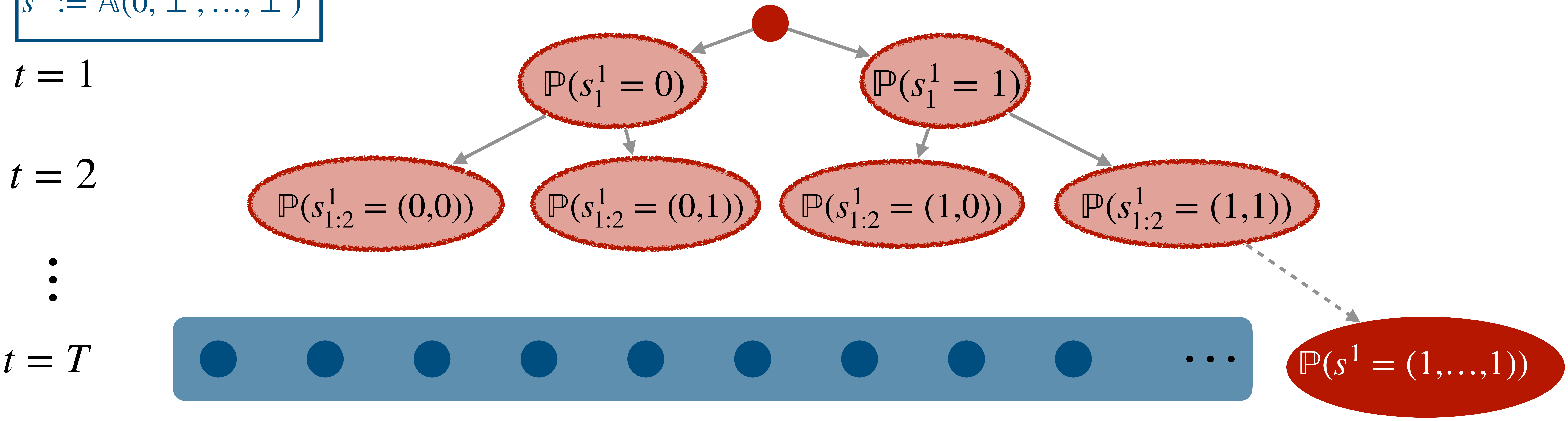

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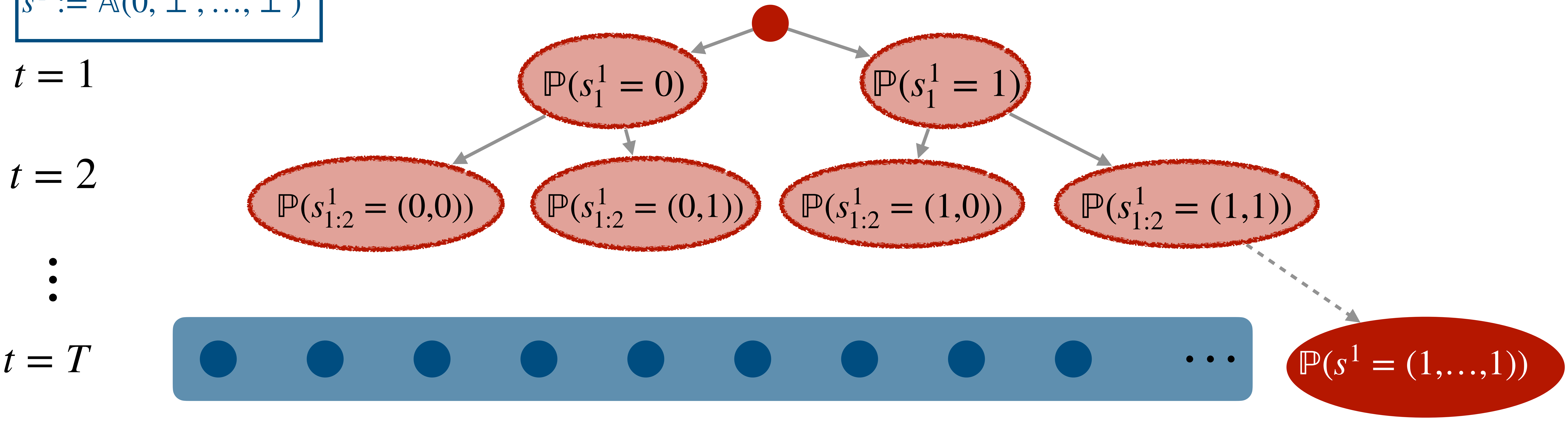

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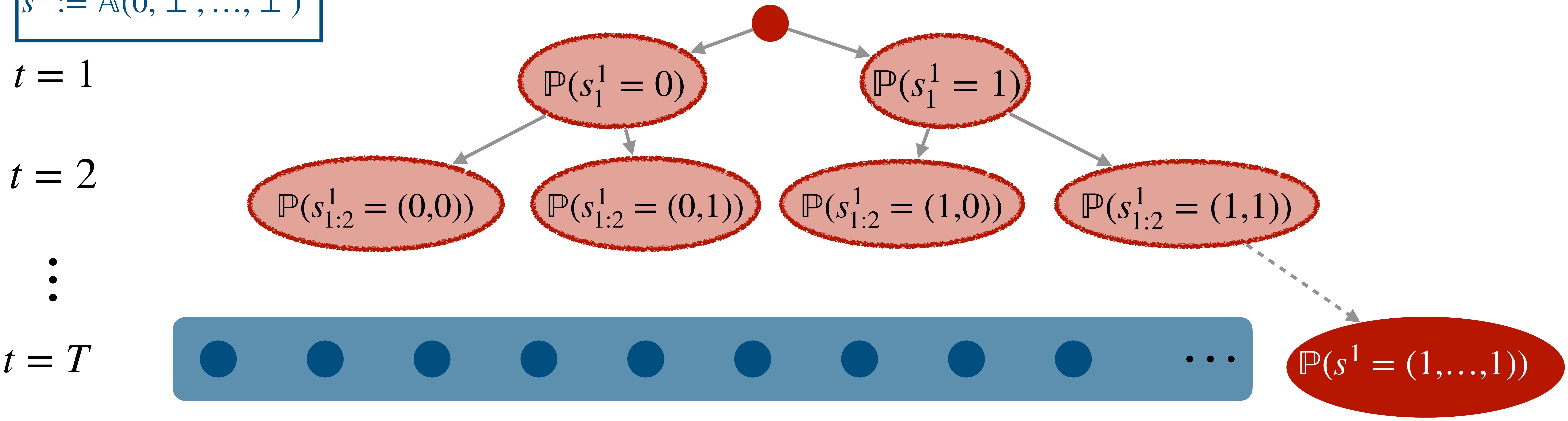

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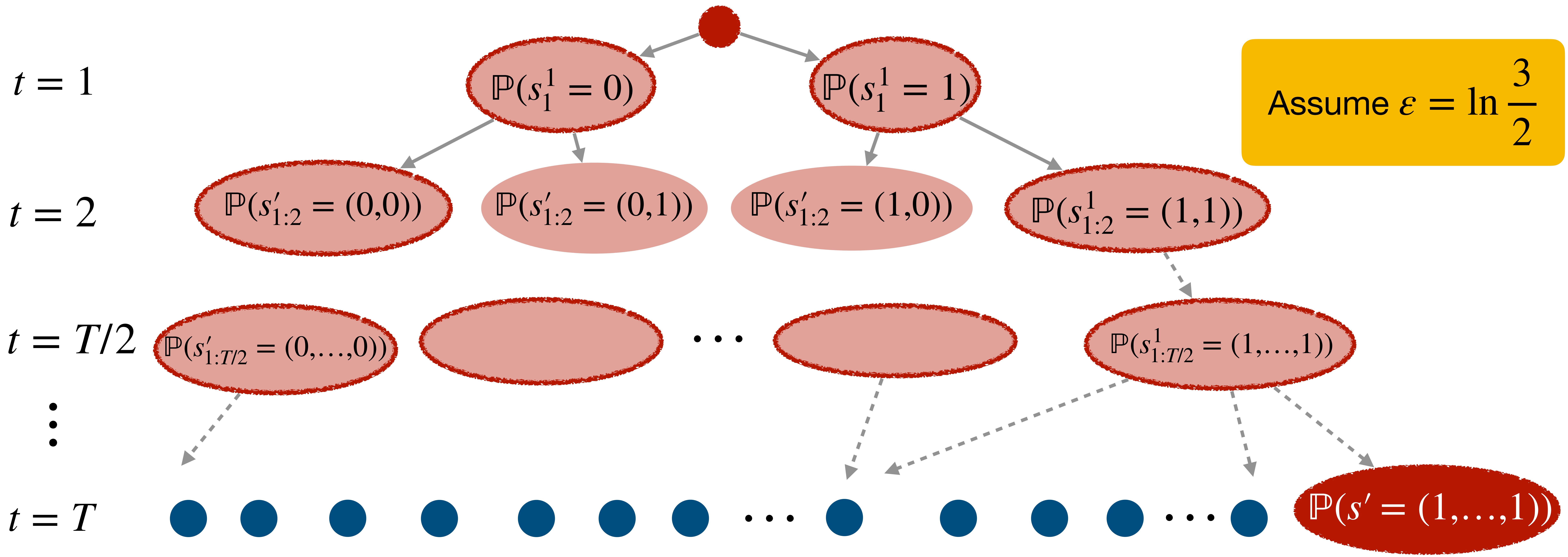
Applying DP on the `correct' set (cont.)



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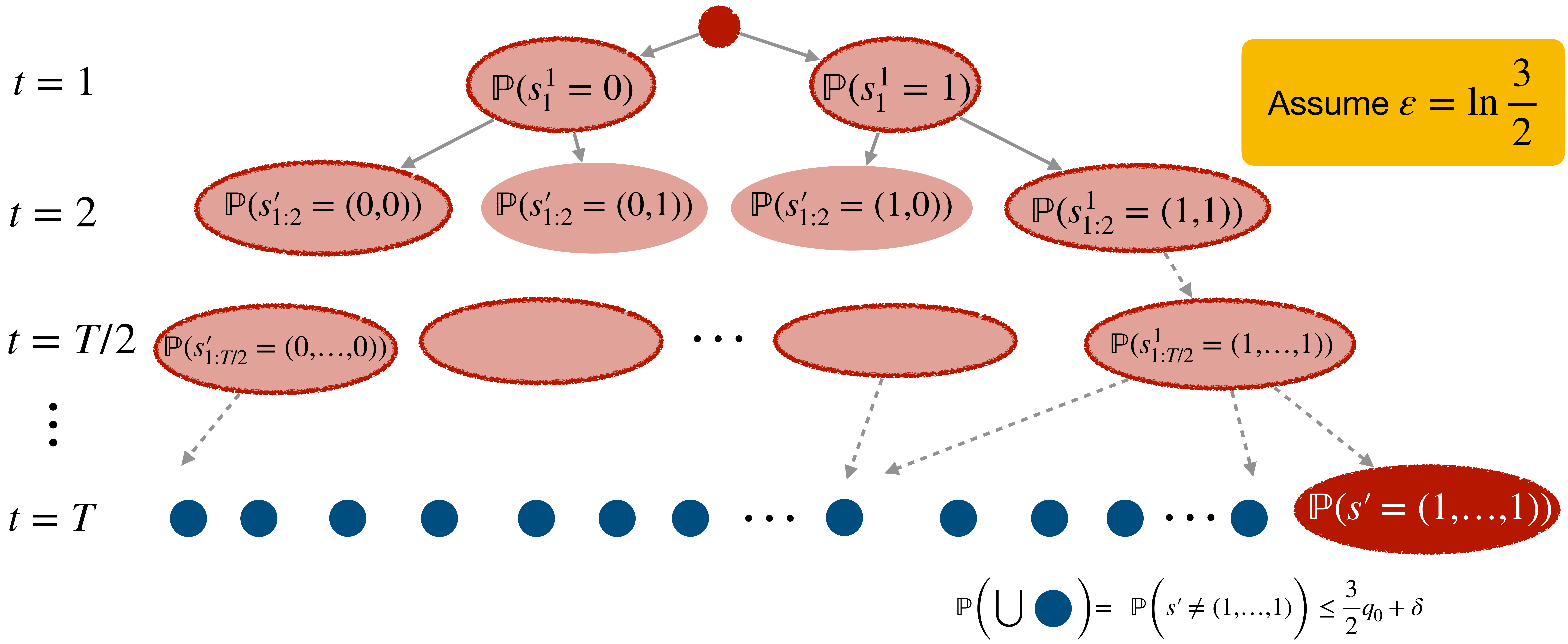

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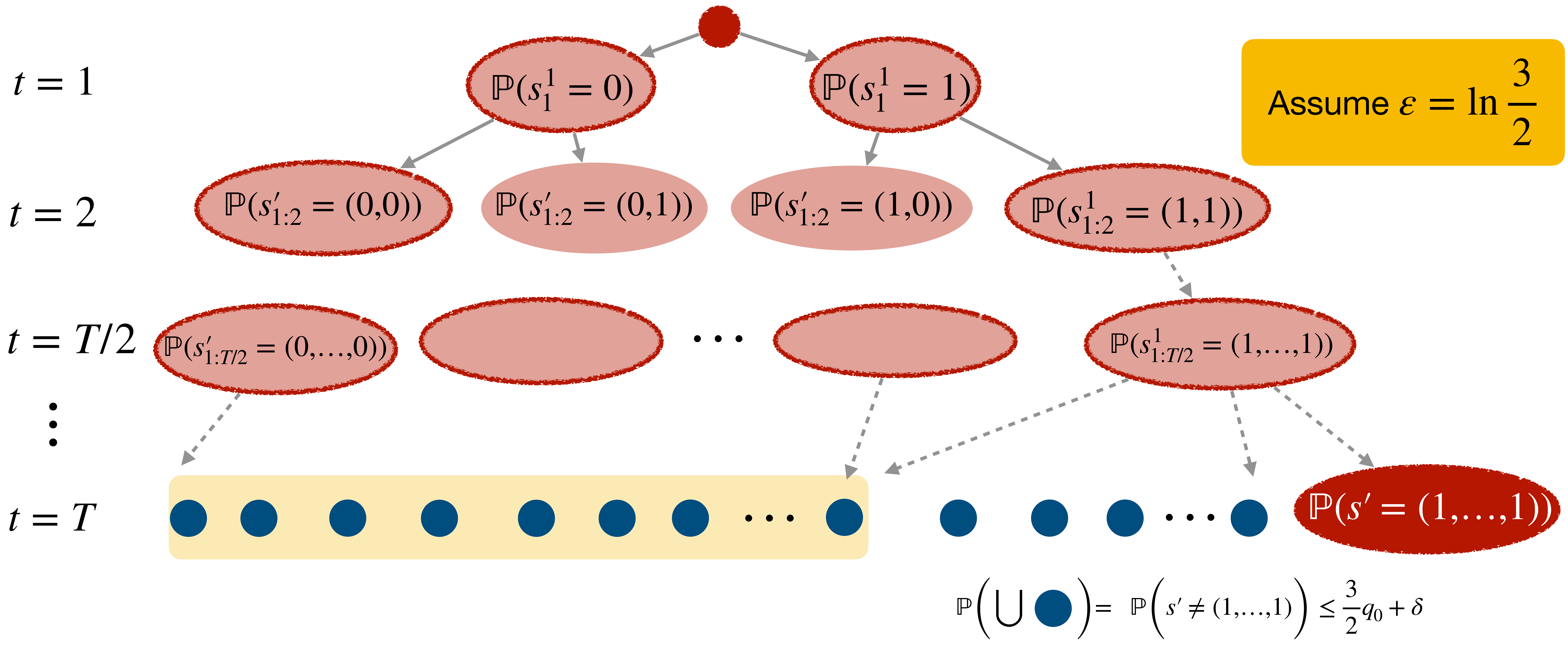

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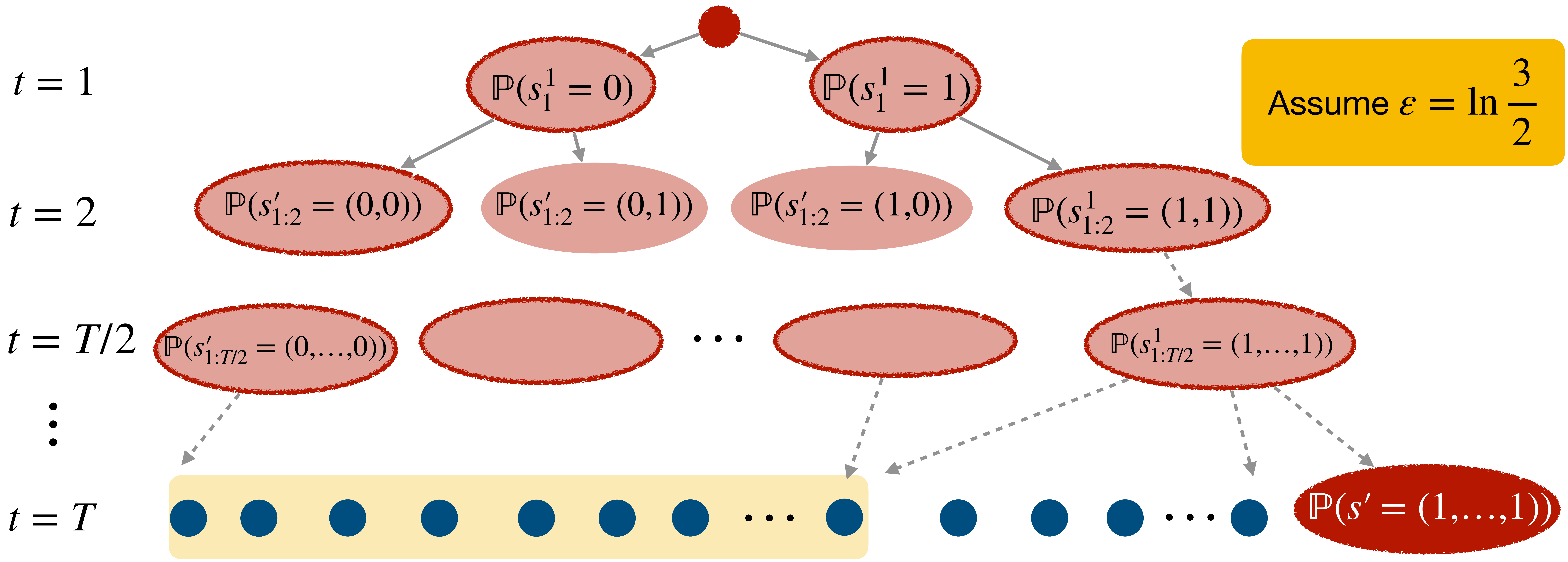

I are the sequences containing '0' at least once before $t = T/2$

$$\mathbb{P}\left(\bigcup \bullet\right) = \mathbb{P}\left(s' \neq (1,\ldots,1)\right) \leq \frac{3}{2} q_0 + \delta$$

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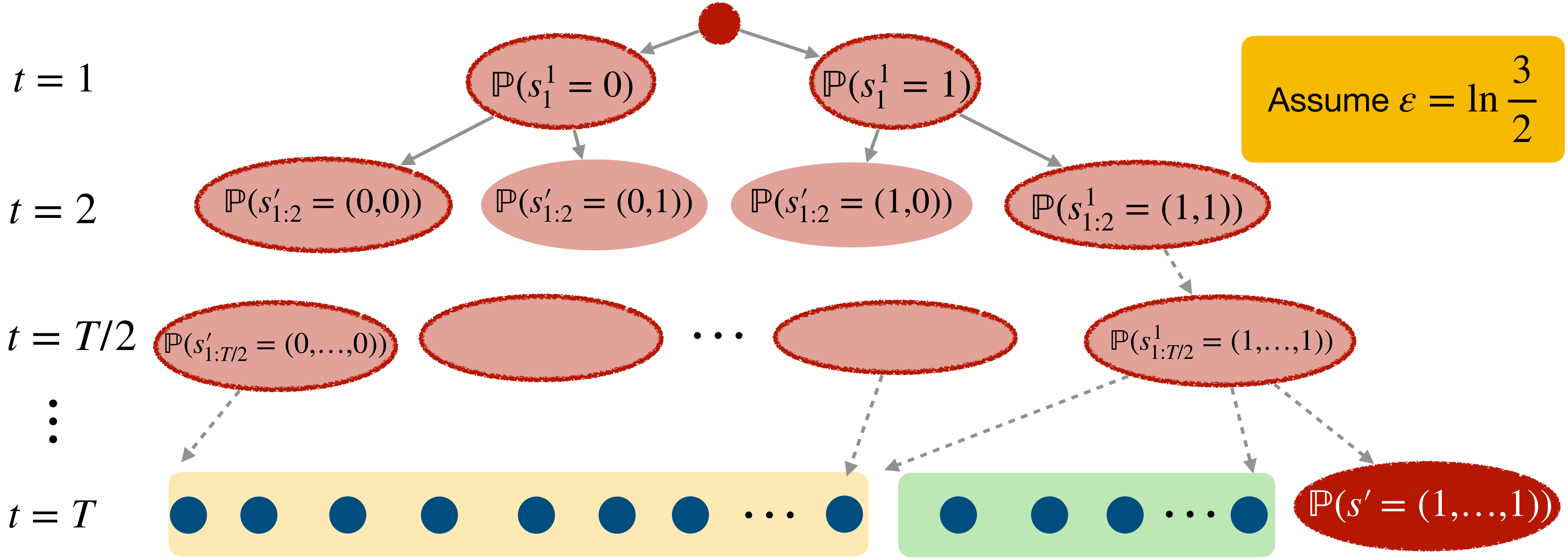

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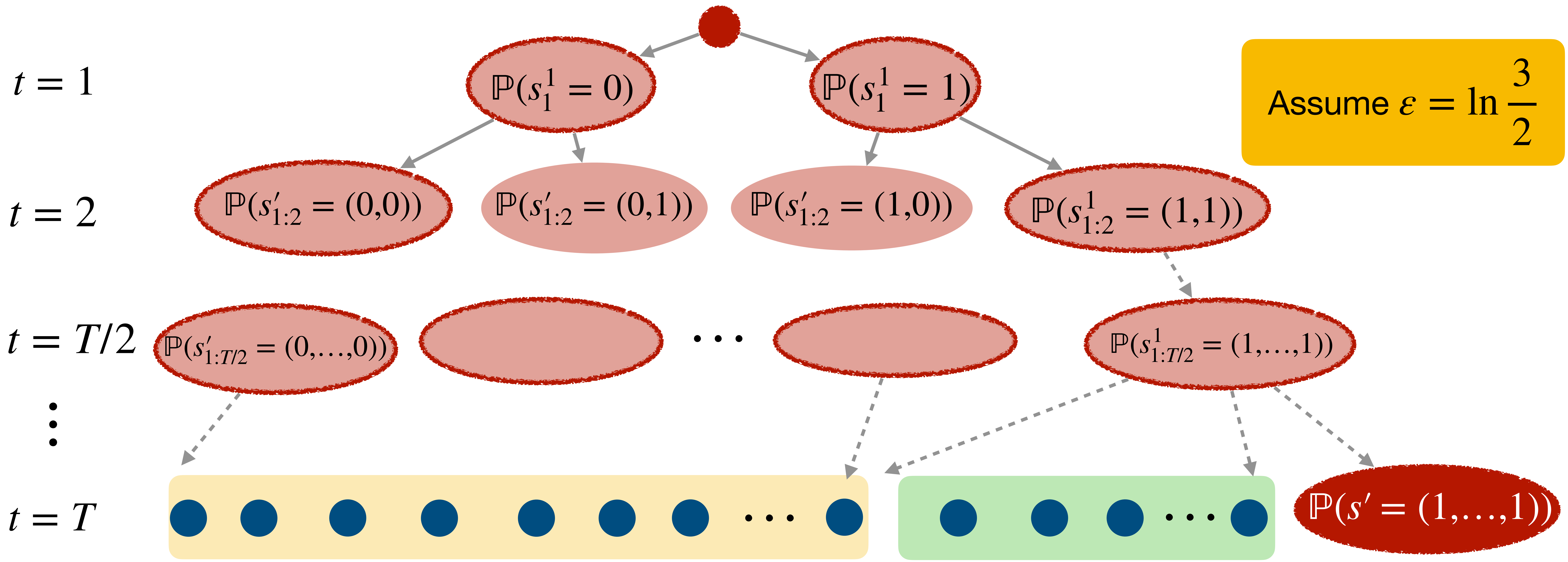

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II are the sequences containing only '1's until $t = T/2$ and containing '0' at least once when $T/2 < t \leq T$

Applying DP on the `correct' set (cont.)

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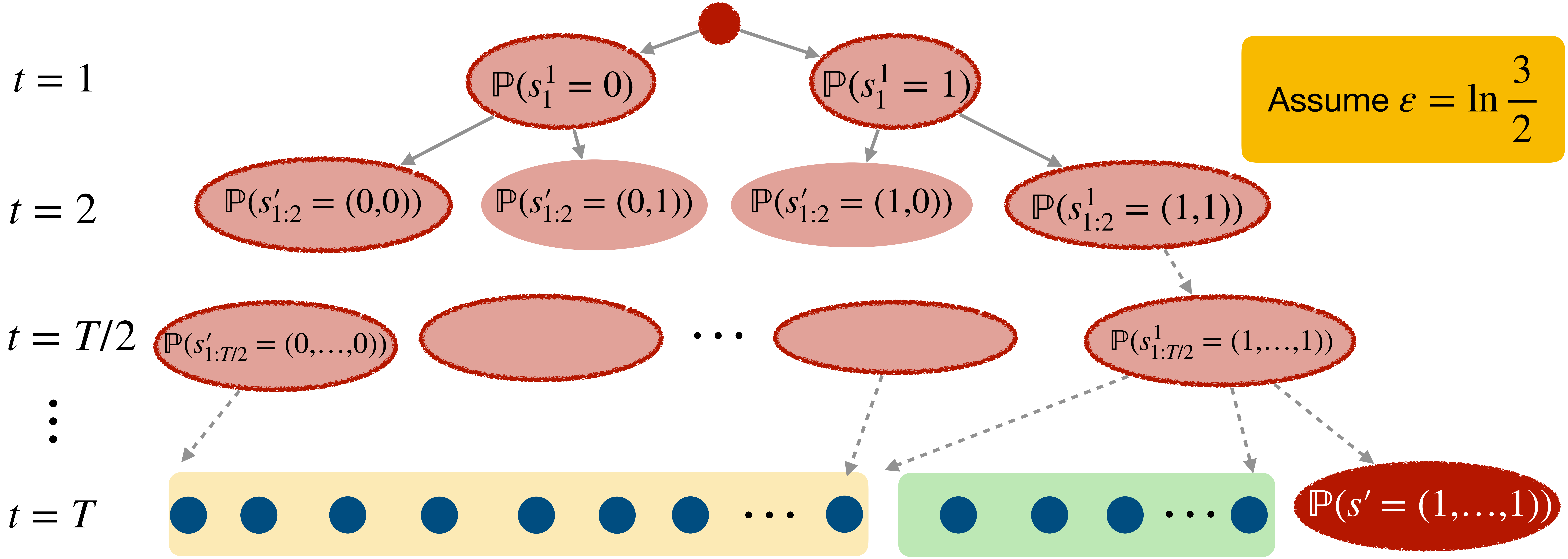

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if $\mathbb{P}\left(\boxed{\text{I}}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2}$, reiterate in the first half and set $q_1 := \mathbb{P}\left(\boxed{\text{I}}\right)$

Recursion

$\boxed{\text{I}}$ are the sequences containing '0' at least once before $t = T/2$

$\boxed{\text{II}}$ are the sequences containing only '1's until $t = T/2$ and containing '0' at least once when $T/2 < t \leq T$

$$\mathbb{P}\left(\boxed{\text{I}}\right) + \mathbb{P}\left(\boxed{\text{II}}\right) \leq \frac{3}{2}q_0 + \delta \quad \Longrightarrow \quad \mathbb{P}\left(\boxed{\text{I}}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2} \quad \text{or} \quad \mathbb{P}\left(\boxed{\text{II}}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2}$$

if $\mathbb{P}\left(\boxed{\text{I}}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2}$, reiterate in the first half and set $q_1 := \mathbb{P}\left(\boxed{\text{I}}\right)$

if $\mathbb{P}\left(\boxed{\text{II}}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2}$, reiterate in the second half and set $q_1 := \mathbb{P}\left(\boxed{\text{II}}\right)$

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$\boxed{\mathrm{I}}$ are the sequences containing ‘0’ at least once before $t = T/2$

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$$\mathbb{P}\left(\mathrm{I}\right) + \mathbb{P}\left(\mathrm{II}\right) \leq \frac{3}{2}q_0 + \delta \implies \mathbb{P}\left(\mathrm{I}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2} \quad \text{or} \quad \mathbb{P}\left(\mathrm{II}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2}$$

if $\mathbb{P}\left(\mathrm{I}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2}$, reiterate in the first half and set $q_1 := \mathbb{P}\left(\mathrm{I}\right)$

if $\mathbb{P}\left(\mathrm{II}\right) \leq \frac{3}{4}q_0 + \frac{\delta}{2}$, reiterate in the second half and set $q_1 := \mathbb{P}\left(\mathrm{II}\right)$

If we continue, we get $q_{i+1} \leq \frac{3}{4}q_i + \frac{\delta}{2} \implies q_i \leq 2\delta$

Constructing hard sequence

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$\mathbb{P}(\mathbb{A}(s^{(k)})$ outputs a 0 in the k^{th} blue blocks$) = q_k \leq 2\delta$

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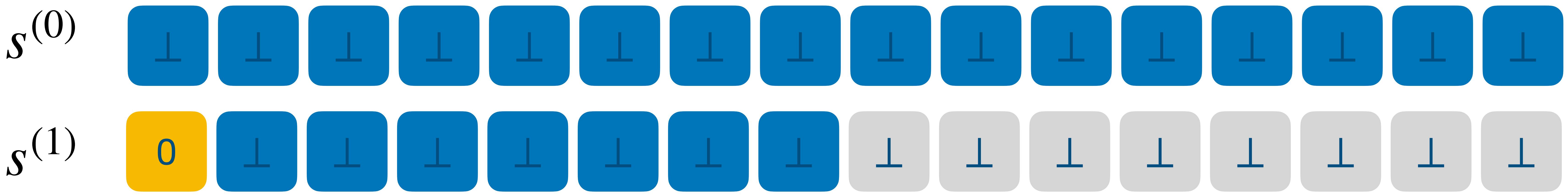

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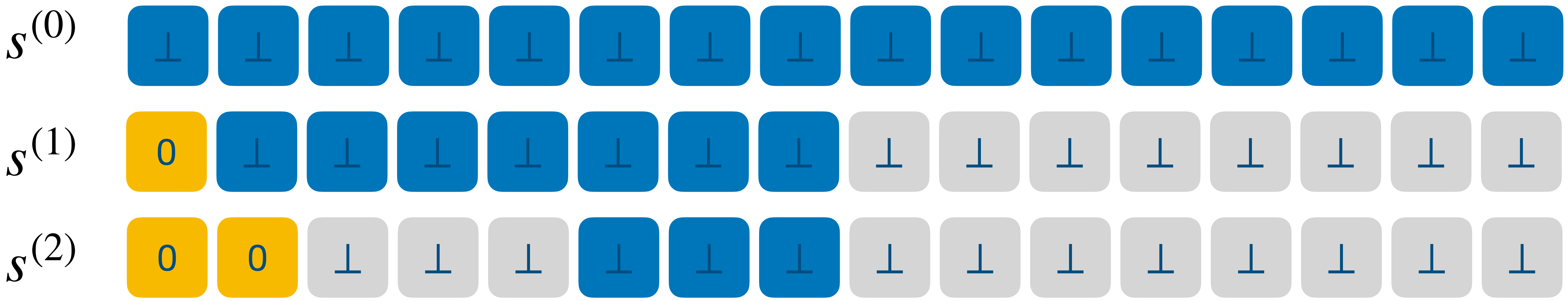

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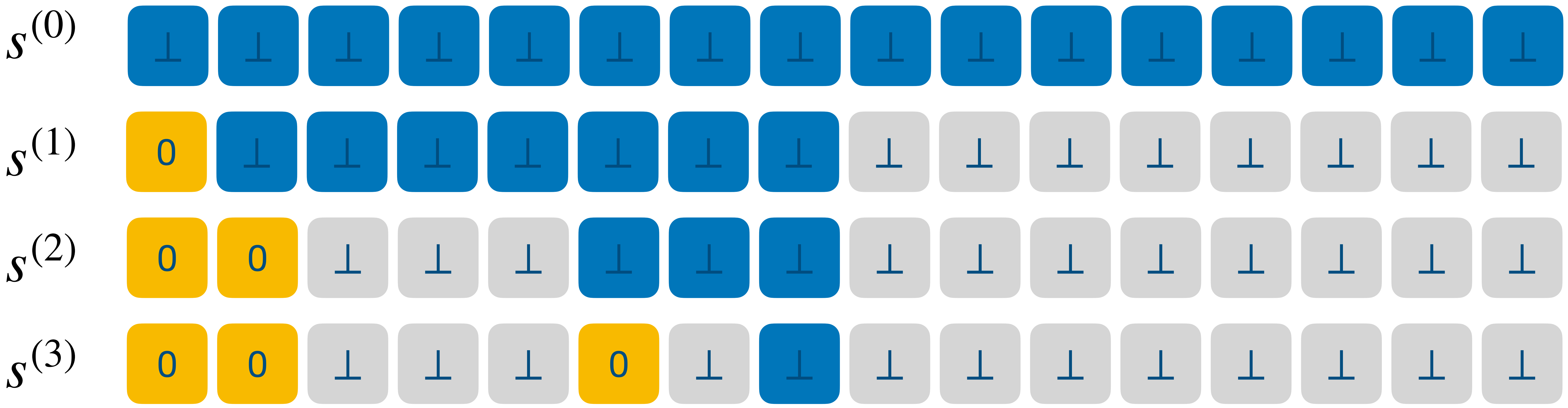

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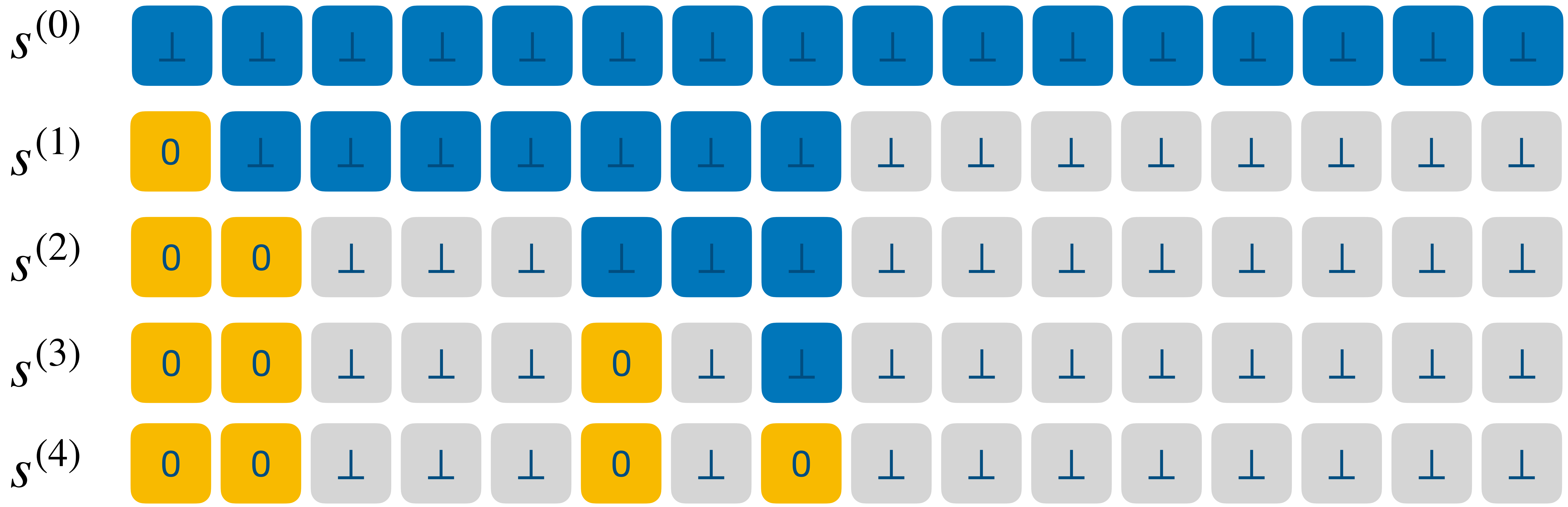

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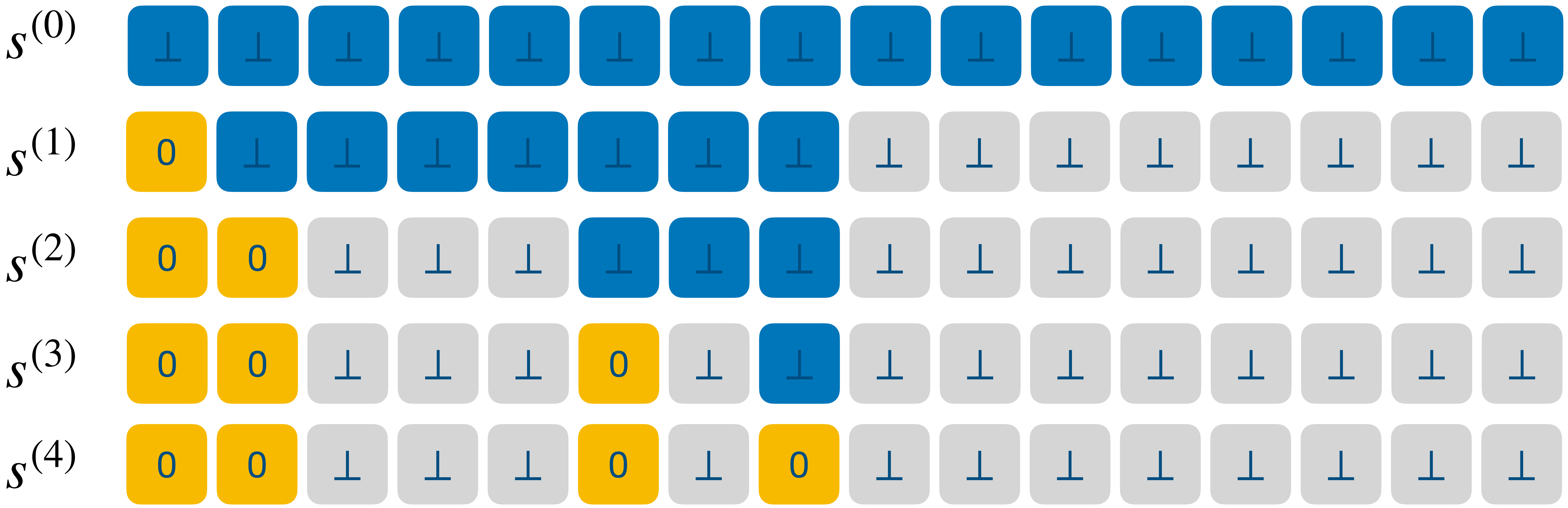

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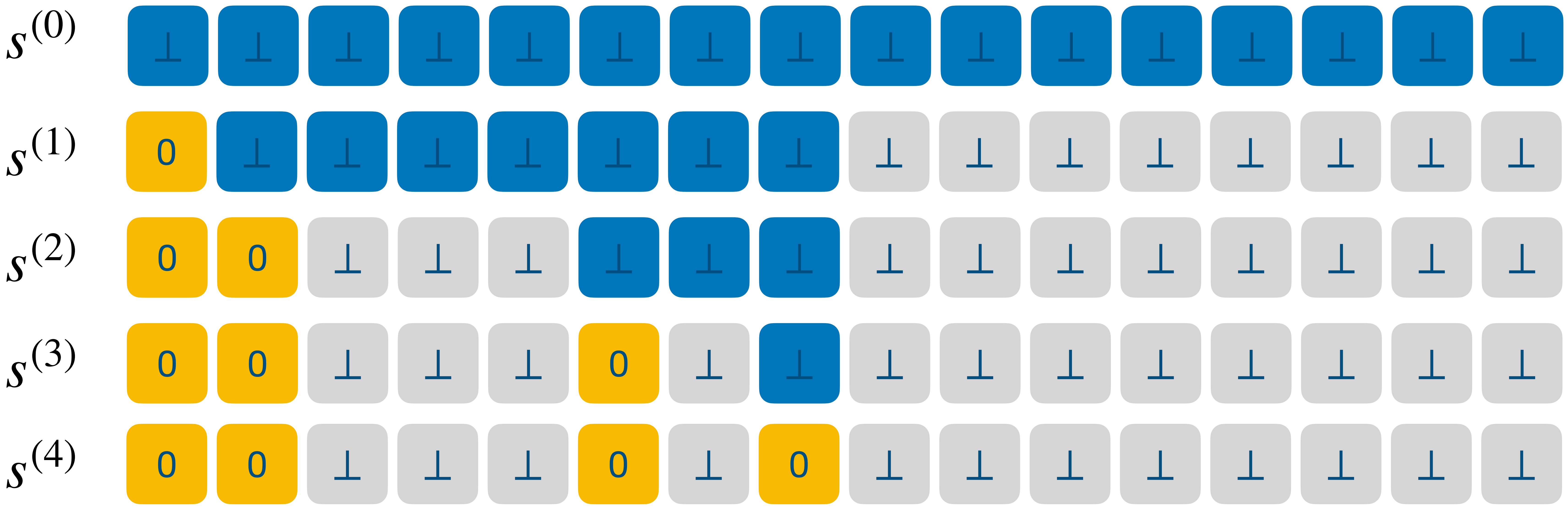

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$\leq \mathbb{P}(\exists i^{\text{th}}$ blue blocks, such that $\mathbb{A}(s^{(i)})$ contains $0)$

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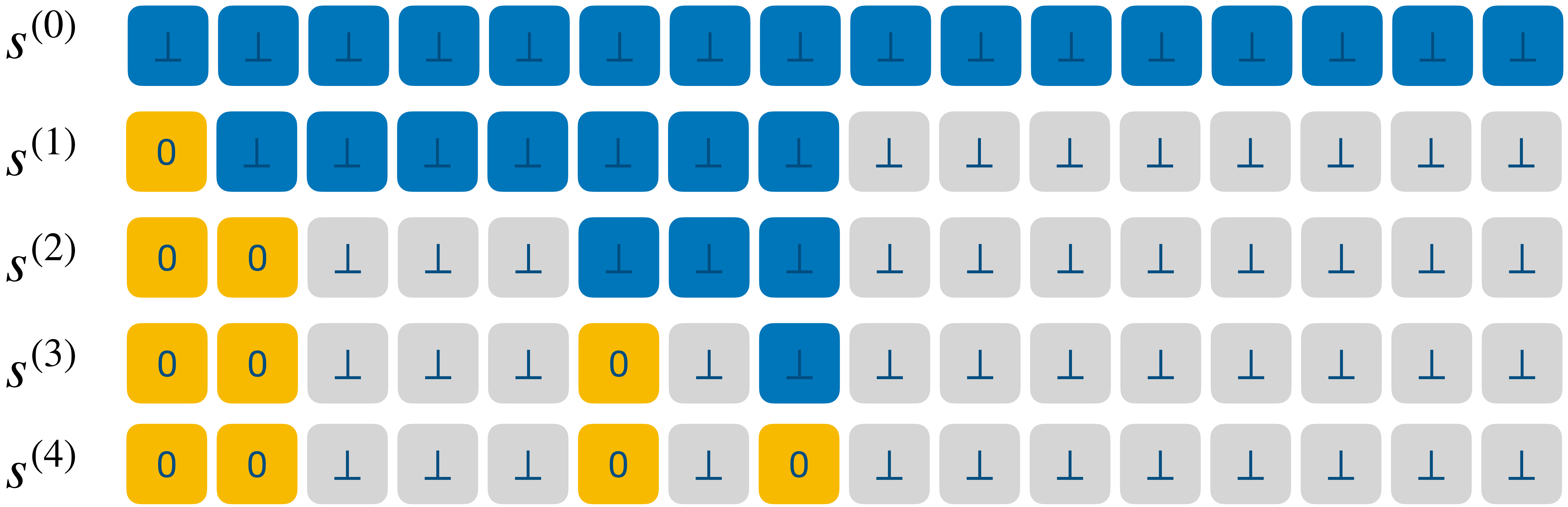

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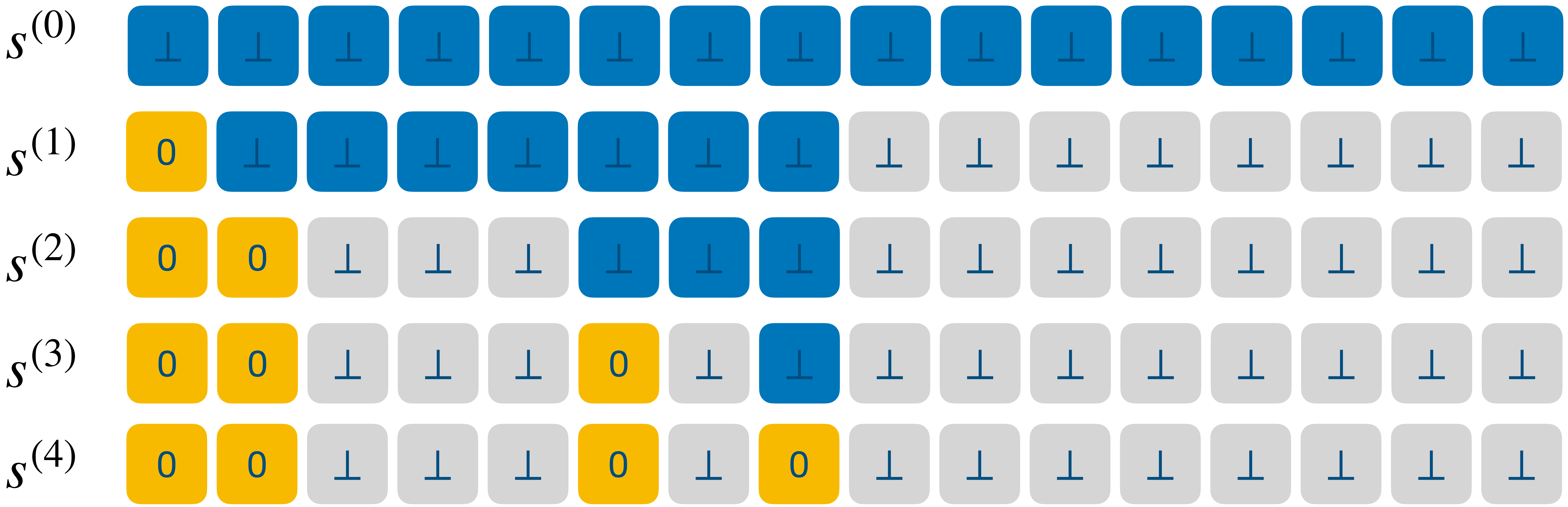

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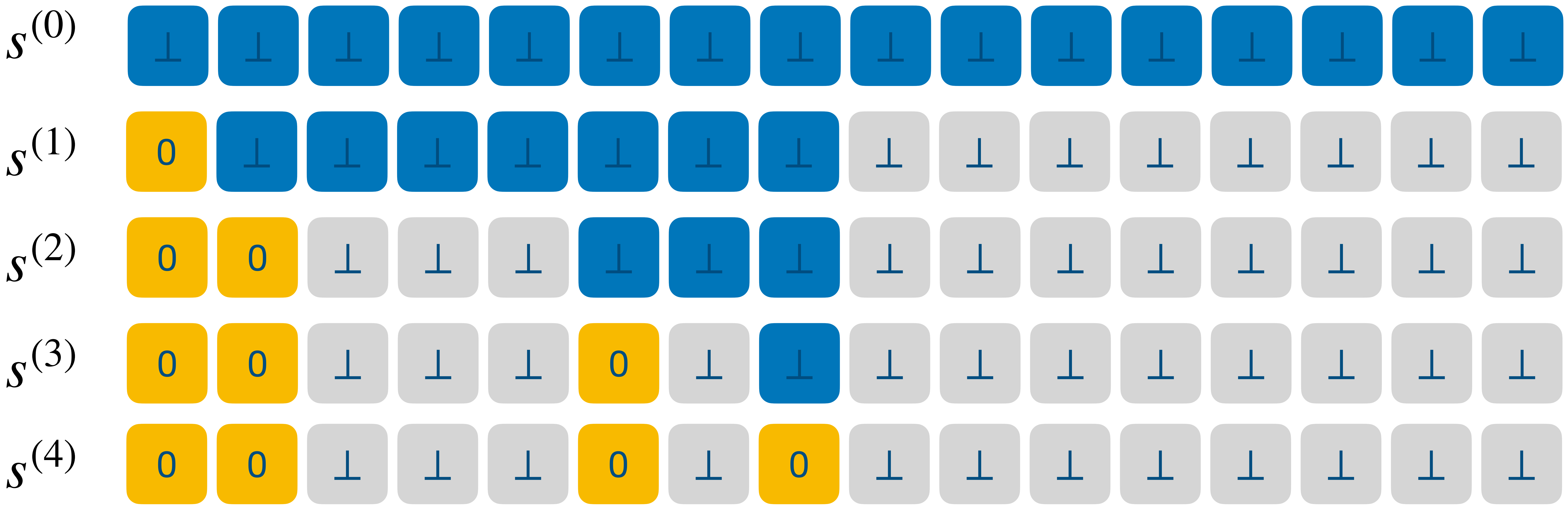

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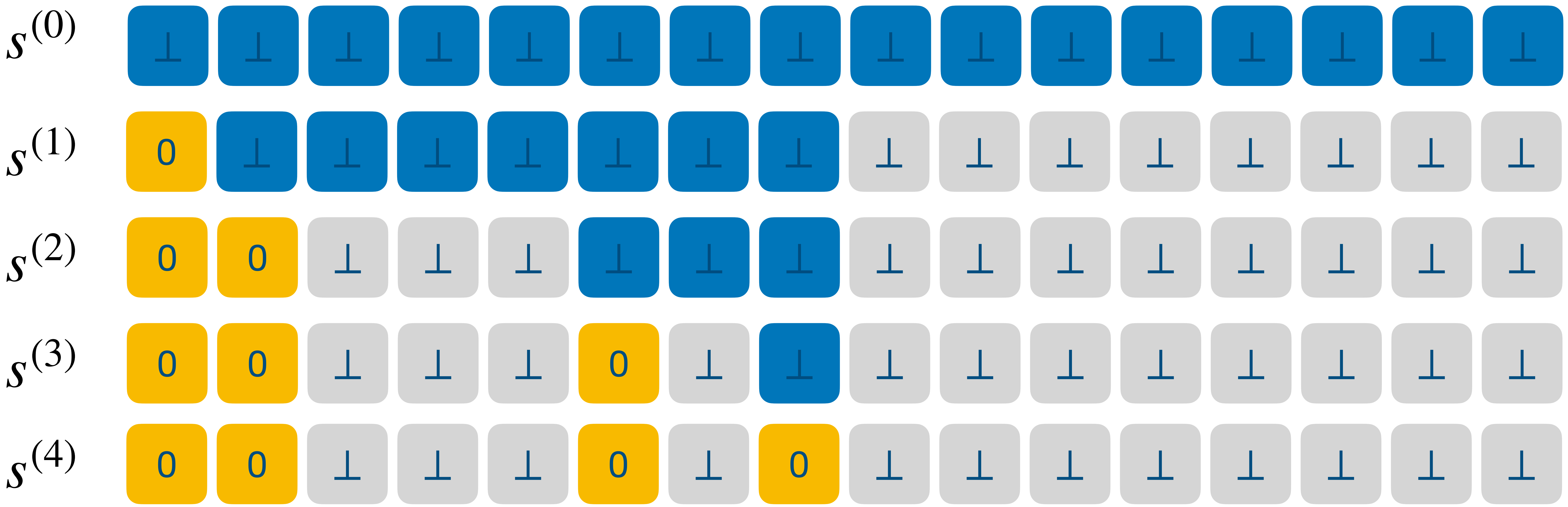

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As long as $k \leq \frac{1}{2}\log T \leq \frac{1}{4\delta}$

$$\implies \mathbb{E}\,[\mathrm{M}] \geq \min\left(\frac{\log T}{4}, \frac{1}{4\delta}\right)$$

Takeaways and open problems

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Lower bounds

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Our contribution
for concentrated (and uniform) algorithms

Takeaways and open problems

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For any algorithm
[CLNSS24]

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Takeaways and open problems

Conjecture
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Conjecture (For any algorithm)

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Our contribution for concentrated (and uniform) algorithms

For any algorithm [CLNSS24]

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Via Reduction
To DP continual observation

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For any algorithm
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Takeaways and open problems

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Thank you

If you are interested to work on questions like these,

I am looking to advise

- PhD & postdocs in Copenhagen and
- UGPs in IIT Kanpur.